



NORGES HANDELSHØYSKOLE

BERGEN, June 17, 2008

Capital Structure - Bankruptcy and Liquidation

by

Kjetil Hollund

Advisor: Svein-Arne Persson

Master Thesis in Economic Analysis

NORGES HANDELSHØYSKOLE

This thesis was written as a part of the masterprogram at NHH. Neither the institution, the advisor, nor the sensors are - through the approval of this thesis - responsible for neither the theories and methods used, nor results and conclusions drawn in this work.

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1 Introduction

In recent years there has been published articles focusing on capital structure in continuous time modeling. These articles have been focusing on various issues that arises when optimizing the capital structure in a firm. To some extent new issues arises as the articles focuses on solving other issues. These articles can be separated into two main categories. Static models and dynamic models. The difference is whether the capital structure is optimized once or if it is continuously optimized as time goes by.

All these models have to some degree been compared to the static capital structure model presented in Leland (1994). This model has some really nice interpretations which makes it easy to understand and use the closed form solutions found within the model. Still it addresses many of the different issues that arises when dealing with these type of models. However, this model has some rather strong limitations. First of all, it is only a static model. Leverage is optimized initially and debt is not restructured as time goes by. Second, this model does not differentiate between bankruptcy and liquidation. Here bankruptcy refers to what is known as bankruptcy according to chapter 11, where a debtor stops paying the creditor the agreed combination of interest and down payment. Liquidation refers to what is known as bankruptcy according to chapter 7, where there is a change of ownership and control over the debtors assets. For this thesis, the assets is the firm itself. When Leland (1994) does not allow for this differentiation, value might be destroyed, and firms that are profitable in the long run might be sold off and terminated. A second problem is the social aspect that arises when firms are liquidated in terms of people loosing their jobs.

Hart (2000) addressed the need for some goals in setting up a good bankruptcy procedure. He identified three goals that should be satisfied when setting up a bankruptcy procedure. Broadie, Chernov, and Sundaresan (2007) used these goals in their article when they expanded the results of Leland (1994), where they separated bankruptcy and liquidation. Checking the solutions provided by these two models, Leland (1994) gives closed form solutions, while Broadie et al. (2007) needs to be solved numerically. The difficult methodology used for solving the latter model alongside with the model being a static model the largest limitations of this model.

The goals for this thesis is therefore to use the economic framework built in Broadie et al. (2007) to modify Leland (1994), but keep the solutions on closed form in contrast to Broadie et al. (2007). The reason for this is that closed form solutions are easier both to understand and to use. This thesis will therefore also focus on explaining the model intuitively as it is developed. In order to keep the model as simple as possible and to be

able to find closed form valuations of claim in the firm, all cash flows will be modeled as perpetual annuities. The main goal will be to set the liquidation and bankruptcy barrier in such a way that the goals provided in Hart (2000) are satisfied and the results from Leland (1994) is closer linked to the observations found in the real world.

Section 2 will set up the model from Leland (1994). The model will be slightly changed such that the underlying value process is the earnings in the firm contrary to the value of the activities used originally. The modified model will be presented thoroughly. This is due to the modified Leland (1994) model will be used as a benchmark model for the new model that will be developed in sections 4 and 5. Therefore it is necessary to give the reader a good understanding of the benchmark model, and hopefully the intuitive explanations of the new model will become clearer.

The next section, section 3, will set up the necessary results provided by Mjøs and Persson (2008) and Mjøs, Persson, and Huang (2008). These results will be treated as pure mathematical formulas, hence there will not be a discussion whether these results hold. The results are based on the exact same underlying value process as will be presented in section 2.1, and they are applicable to the world described in Broadie et al. (2007).

In section 4 the basics in the new model will be presented. Troublesome issues will be highlighted, and possible solutions will be discussed. The main focus for this section will be to check if the economical framework from Broadie et al. (2007) can be applied to Leland (1994) by using the annuities from Mjøs and Persson (2008).

Section 5 will fully derive the new model and thoroughly present some comparative statics. The results will be explained and compared to the Leland (1994) model. The model will be based upon the economic framework set up in Broadie et al. (2007) and it will be coherent with the goal identified in Hart (2000).

Finally, section 6 will add some closing remarks about the model developed and how it perform compared to the Leland (1994) model.

2 Modified Leland (1994)

Starting off, this section will introduce the economical framework set up in Leland (1994). Leland's original model will be slightly changed such that the underlying value process is an EBIT stream generated by the firm assets, instead of the value of the firm's activities. This EBIT stream, which is following a standard stochastic process, was used in Goldstein, Ju, and Leland (2001).

2.1 The Value Process

In Goldstein et al. (2001) the authors assume that the firm's assets generate an EBIT cash flow denoted δ_t , given by the stochastic process

$$d\delta_t = \mu\delta_t dt + \sigma\delta_t dW_t. \quad (2.1)$$

Here, the drift and the volatility, denoted μ and σ respectively, are constants, and δ_0 is the fixed initial cash flow level.

The time t market value, denoted V_t , of the assumed perpetual EBIT stream from the assets equals

$$V_t = E_t^Q \left[\int_0^\infty e^{-r(s-t)} \delta_s ds \right] = \frac{\delta_t}{r - \mu}. \quad (2.2)$$

Hence, if we substitute the market value V_t from equation (2.2) into equation (2.1), the market value, V_t , is given by the stochastic process

$$\begin{aligned} dV_t &= (rV_t - \delta_t)dt + \sigma V_t dW_t \\ &= \mu V_t dt + \sigma V_t dW_t. \end{aligned} \quad (2.3)$$

A claim on the firm's EBIT stream as a function of V and t , denoted $F(V, t)$, continuously pays a non-negative coupon, c , as long as the firm is solvent. According to Merton (1974), $F(V, t)$ must therefore satisfy the fundamental partial differential equation

$$\frac{1}{2}\sigma^2 V^2 F_{VV}(V, t) + \mu V F_V(V, t) + F_t(V, t) - rF(V, t) + c = 0. \quad (2.4)$$

This equation has, in general, no closed form solution. However, a closed form solution can be found by considering perpetual claims to the EBIT stream. Then the term $F(V, t) = 0$, and equation (2.4) is changed to the ordinary differential equation

$$\frac{1}{2}\sigma^2 V^2 F''(V) + \mu V F'(V) - rF(V) + c = 0. \quad (2.5)$$

In order to solve this equation, we first consider the homogenous part

$$\frac{1}{2}\sigma^2 V^2 F''(V) + \mu V F'(V) - rF(V) = 0. \quad (2.6)$$

By inserting $F(V) = V^X$ into equation (2.6), we get

$$\frac{1}{2}\sigma^2 V^2 X(X-1)V^{X-2} + \mu V X V^{X-1} - r V^X = 0. \quad (2.7)$$

Assume V is always positive, hence V^X is also always positive. Then we can divide by V^X , and we are left with

$$\frac{1}{2}\sigma^2 X(X-1) + \mu X - r = 0. \quad (2.8)$$

Solving for X , we get the solutions $x_1 = \alpha$ and $x_2 = -\beta$

$$\alpha = \frac{\frac{1}{2}\sigma^2 - \mu + \sqrt{(\frac{1}{2}\sigma^2 - \mu)^2 + 2\sigma^2 r}}{\sigma^2}, \quad (2.9)$$

and

$$\beta = \frac{\mu - \frac{1}{2}\sigma^2 + \sqrt{(\frac{1}{2}\sigma^2 - \mu)^2 + 2\sigma^2 r}}{\sigma^2}. \quad (2.10)$$

Here α and β satisfies the inequalities $\alpha > 1$ and $\beta > 0$.

The general solution of the homogenous part of equation (2.5) is therefore

$$F^H(V) = K_1 V^\alpha + K_2 V^{-\beta}, \quad (2.11)$$

and the corresponding general solution of equation (2.5) is

$$F(V) = K_0 + K_1 V^\alpha + K_2 V^{-\beta}, \quad (2.12)$$

where K_0 , K_1 , and K_2 are some constants.

2.2 Value of Debt

The firm, with an EBIT stream as described above, adds debt to its capital structure. The debt promises a perpetual coupon payment, c , and the value of this perpetual claim is denoted $D(V)$. The coupon remains constant, as long as the firm is solvent. In this model bankruptcy and liquidation are not separated, in other words they happen at the exact same time. Let V_B denote the level where bankruptcy is declared. This happens when the firm stop paying coupons. In bankruptcy, a fraction $0 \leq \lambda \leq 1$ of the value, V , will be lost in bankruptcy costs. The creditors are then left with $(1 - \lambda)V_B$ while the share holders get nothing.

Searching for boundary conditions, the value of debt must satisfy the following conditions:

$$D(V) = (1 - \lambda)V_B, \text{ when } V = V_B, \quad (2.13)$$

and

$$D(V) \rightarrow \frac{c}{r}, \text{ as } V \rightarrow \infty. \quad (2.14)$$

The first condition says that the value of debt is equal to the remaining value of the firm after bankruptcy costs at bankruptcy. The second condition says that when V gets very high, the value of debt converges to the value of risk free debt. The reason for this is that the probability of default goes towards zero as V goes to infinity.

Applying these boundary conditions to the general solution found in equation (2.12), one can find the constants K_0 , K_1 , and K_2 . Applying equation (2.14) to equation (2.12), gives $K_1 = 0$. This is to keep the value of debt from increasing exponentially as V increases. Also, K_0 can be found by observing that as $V \rightarrow \infty$, $V^{-\beta} \rightarrow 0$, hence $K_0 = \frac{c}{r}$. Finally, applying equation (2.13) to equation (2.12), K_2 is found to be $K_2 = [(1 - \lambda)V_B - \frac{c}{r}] V_B^\beta$. The value of debt is therefore given by

$$D(V) = \frac{c}{r} + \left[(1 - \lambda)V_B - \frac{c}{r} \right] \left[\frac{V}{V_B} \right]^{-\beta}. \quad (2.15)$$

$D(V)$ is increasing in the parameters c and V , and decreasing in the parameters r , λ , and V_B .

This expression has some useful interpretations. The first term is the value of risk free debt. The second term has two parts. The first part explains what happens at bankruptcy. There the creditor lose the value of the risk free debt and gain the liquidation value of the firm after bankruptcy costs. The second part can be interpreted as the price of a claim, denoted U^a , which pays 1 the first time V hits the bankruptcy barrier V_B from above.

$$U^a = \left[\frac{V}{V_B} \right]^{-\beta}. \quad (2.16)$$

Instead of continuing to use the rather tedious procedure described to derive the value of debt, equation (2.16) can be used to value claims directly. This is done by reducing the value of a riskless perpetual annuity by what is lost in case of bankruptcy multiplied by the price of the claim which pays 1 in the case of bankruptcy. This is exemplified in the interpretation of the value of debt. Going forward, this is the method that will be used to value the various claims on the firm.

The debt in place has two implications for the total value of the firm. First, it reduces value due to possible losses in case of bankruptcy. Second, it increases value due to the

tax deductibility of interest payments. Therefore, value of these claims needs to be found, before the value of the firm and the value of the equity can be derived.

2.3 Value of Bankruptcy Costs

If bankruptcy occurs, a fraction λ of V_B is lost in bankruptcy costs. The value of this claim can be found directly by multiplying the bankruptcy costs with the price of default, given in equation (2.16). The value of bankruptcy costs is therefore given by

$$BC(V) = \lambda V_B \left[\frac{V}{V_B} \right]^{-\beta}. \quad (2.17)$$

$BC(V)$ is increasing in the parameters λ and V_B , and decreasing in the parameter V .

2.4 Value of Tax Benefits

Similar as for the value of bankruptcy costs, the value of tax benefits can be derived by using equation (2.16). As long as the firm pays coupons, it can deduct these by the tax rate, denoted τ . The value of these tax deductions can therefore be found by getting the tax deduction as a perpetual claim, less the value of this claim stopping when the firm declares bankruptcy. The value of the tax benefits is therefore given by

$$TB(V) = \frac{\tau c}{r} \left[1 - \left(\frac{V}{V_B} \right)^{-\beta} \right]. \quad (2.18)$$

$TB(V)$ is increasing in the parameters τ , c , and V , and decreasing in the parameters r and V_B .

2.5 Value of Firm

The total value of the firm, denoted $v(V)$, reflects three aspects; the firm's EBIT stream, the bankruptcy costs, and the tax benefits. The value of the firm is therefore given by

$$\begin{aligned} v(V) &= V + TB(V) - BC(V) \\ &= V + \frac{\tau c}{r} \left[1 - \left(\frac{V}{V_B} \right)^{-\beta} \right] - \lambda V_B \left[\frac{V}{V_B} \right]^{-\beta}. \end{aligned} \quad (2.19)$$

$v(V)$ is increasing in the parameters V , τ , and c , and decreasing in the parameters r and V_B .

2.6 Value of Equity

The value of the equity in the firm, denoted $E(V)$, is the residual claim, after all other stake holders are paid. Hence, the value of equity is given by

$$\begin{aligned} E(V) &= v(V) - D(V) \\ &= V - \frac{(1-\tau)c}{r} + \left[\frac{(1-\tau)c}{r} - V_B \right] \left[\frac{V}{V_B} \right]^{-\beta}. \end{aligned} \quad (2.20)$$

$E(V)$ is increasing in the parameters V and τ , and decreasing in the parameters c and V_B . The bankruptcy barrier, V_B , increases $E(V)$ up to a certain point, before it starts to decrease $E(V)$. The share holders objective is always to maximize their value of equity. It will now be interesting to check if V_B can be determined endogenously by the share holders.

2.7 Endogenous Bankruptcy Barrier V_B

If the barrier for bankruptcy, V_B , can be determined by the share holders in the firm, they will maximize their value of equity with respect to V_B . This is done by differentiating equation (2.20) with respect to V_B , setting the derivative equal to zero, and solving for V_B . The barrier for bankruptcy is then given by

$$V_B = \frac{(1-\tau)c}{r} \frac{\beta}{\beta+1}. \quad (2.21)$$

V_B is increasing in the parameters c and β , and decreasing in the parameters τ and r .

Substituting the expression for V_B into the equations for value of debt (2.15), value of firm (2.19), and value of equity (2.20) gives

$$D(V) = \frac{c}{r} \left[1 - \left(\frac{c}{V} \right)^\beta k \right], \quad (2.22)$$

$$v(V) = V + \frac{\tau c}{r} \left[1 - \left(\frac{c}{V} \right)^\beta h \right], \quad (2.23)$$

and

$$E(V) = V - \frac{(1-\tau)c}{r} \left[1 - \left(\frac{c}{V} \right)^\beta m \right]. \quad (2.24)$$

where

$$\begin{aligned}
m &= \left[\frac{(1-\tau)\beta}{r(\beta+1)} \right]^\beta \frac{1}{\beta+1} \\
h &= \left[(\beta+1) + \frac{\lambda(1-\tau)\beta}{\tau} \right] m \\
k &= [(\beta+1) - (1-\lambda)(1-\tau)\beta] m.
\end{aligned}$$

The results from this modified version of Leland (1994) are the exact same results as in the original Leland (1994). The only difference is found in the expression for β . This is because we allow for $r \neq \mu$ and that the underlying value process is changed from the value of the firm's activities to an EBIT stream.

2.8 Comparative Statics

Here, a numerical example will be presented to find optimal capital structure in the firm, and to check how this affects various claims on the firm's cash flow.

The parameters will be set according to Leland (1994), to make comparison between the original and the modified version as simple as possible. The parameters will therefore have the values; $\mu = 3.5\%$, $\sigma = 20\%$, $r = 6\%$, $\lambda = 50\%$, $\tau = 35\%$, $c = 6$, and $V = 100$.

Figure 1 illustrates how all the claims on the firm's EBIT stream changes as the coupon changes. Value of debt, tax benefits, and firm all got local maxima for different values of the coupon. The value of bankruptcy cost is strictly increasing in the coupon and the value of equity is strictly decreasing in the coupon. To really understand how these claims behave, it is necessary to take a closer look at some comparative statics where some of the parameters are changed.

Figure 2 illustrates the effect on the value of debt of changing the volatility. Maximizing the value of debt with a high volatility gives a higher optimal coupon than maximizing the value of debt with a lower volatility. Also, the effect of changing volatility is a change in the curvature where high volatility gives a flatter curve than low volatility.

Figure 3 illustrates the effect on the firm value of changing the volatility. Maximizing the value of the firm gives different optimal coupon levels depending on the underlying volatility for the firm's EBIT stream. For all illustrated volatilities the optimal coupon is within the range of 6 and 7. If the firm's EBIT stream has a high volatility, a higher coupon is needed in order to maximize the value of the firm than if the firm's EBIT stream has a low volatility. Just from looking at this figure, one can conclude that the value of an unlevered firm can increase by about 20% to 30%, depending on the volatility, by adding debt to the capital structure.

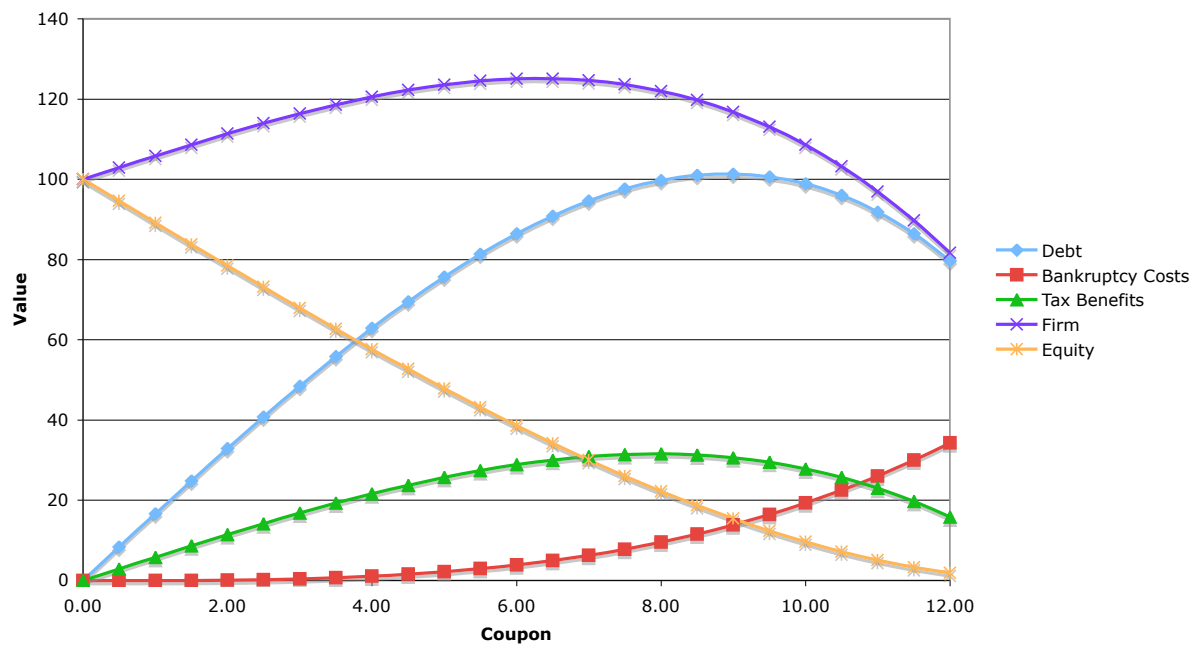


Figure 1: The effect of the coupon on the various claims on the firm.

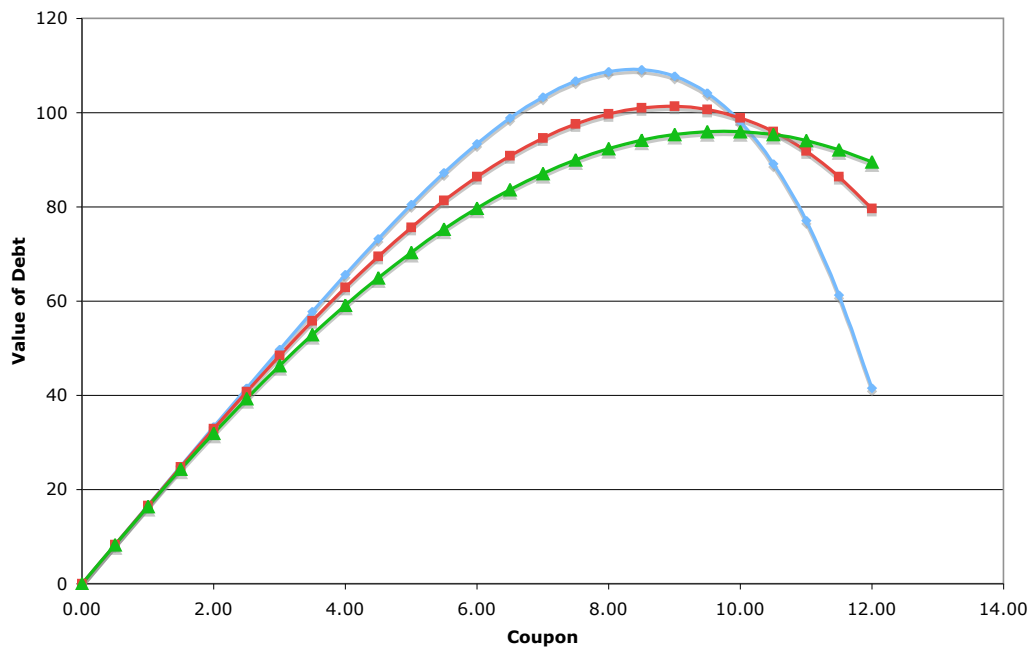


Figure 2: The effect of volatility on the value of debt.

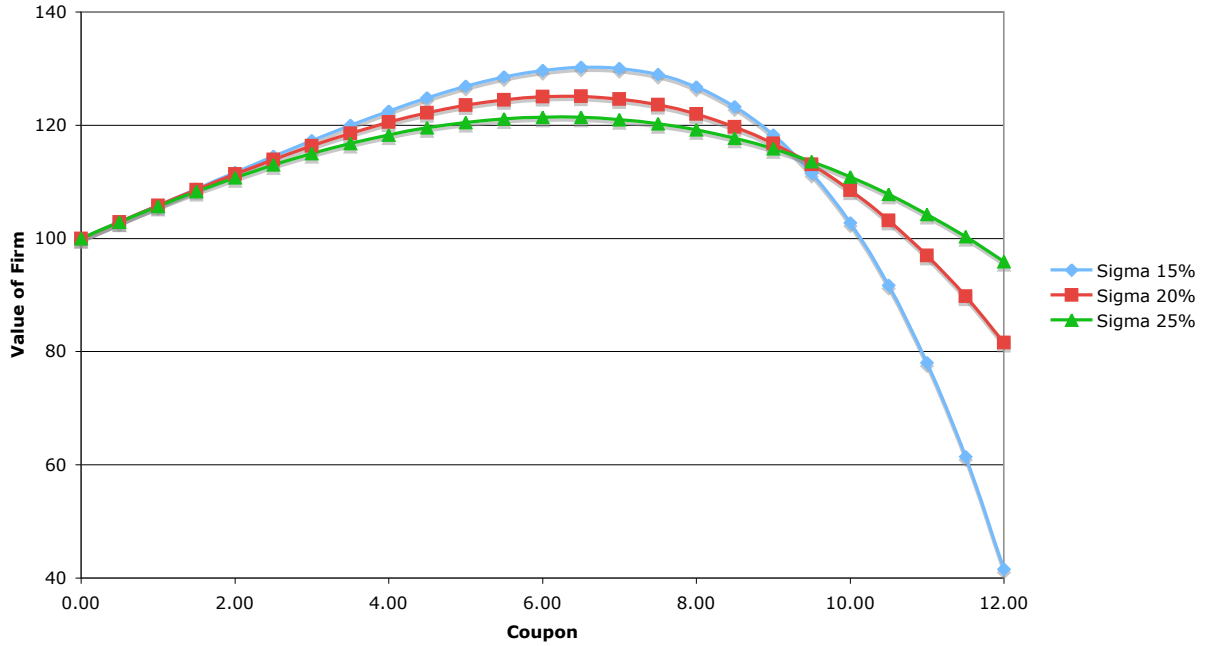


Figure 3: The effect of volatility on the value of the firm.

Figure 4 shows how the value of the firm change as the leverage change. Leverage is found by dividing the value of debt by the value of the firm. Contrary to figure 3 where the value of the firm is maximized at the coupon level, this figure gives higher values for the value of the firm with low volatility at any fixed level of leverage comparing to a higher volatility.

Finally, figure 5 shows how the value of equity changes as the coupon changes. The value of equity is decreasing in the coupon. On the other hand, it is increasing in the volatility. These two properties are the same properties as for a call option, where the value is decreasing in the strike price and increasing in the volatility. Also, since equity is the residual claim on the firm's EBIT stream, equity in fact can be considered as a call option on the firm's EBIT.

As shown in the above, the Leland (1994) model is a static capital structure model which optimizes when the equity holders should stop paying coupons and declare bankruptcy. The limitations of this model is that it only optimizes the capital structure once, and that it does not allow for debt restructuring or refinancing. One can also argue that the tax benefits might be inaccurate. This is because the firm only get a tax benefit if the firm is in a taxable position. But this might not be the case. If the firm is not able to get all the tax benefits, the optimal capital structure will obviously be set differently.

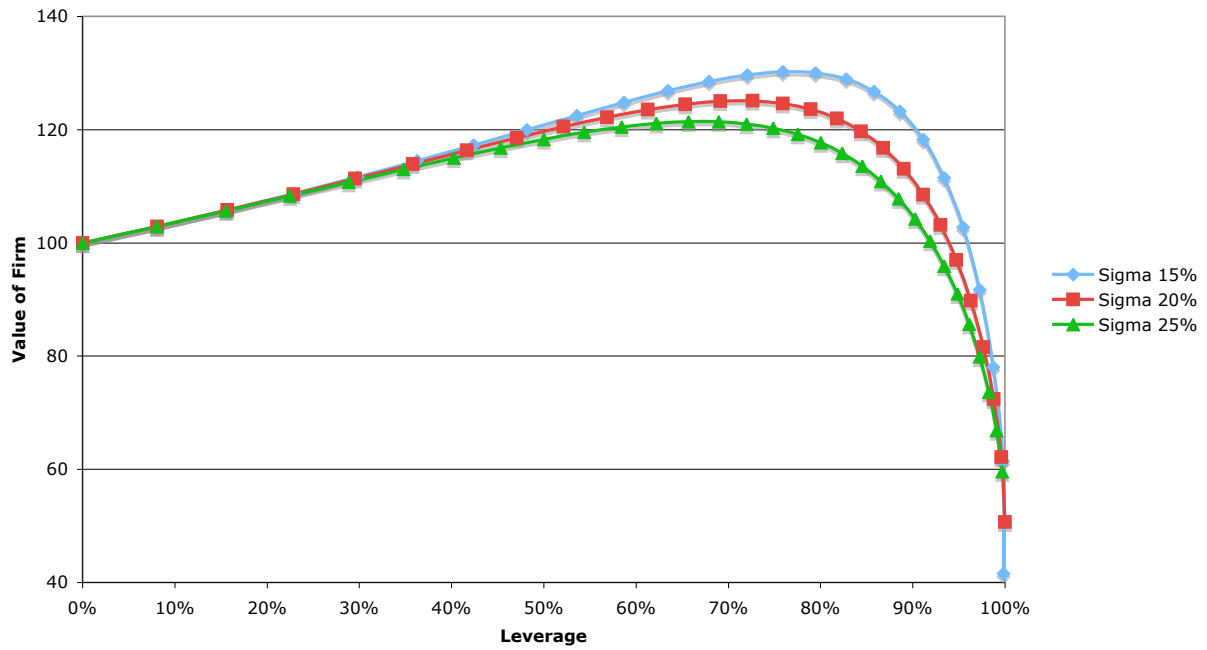


Figure 4: The effect of volatility on leverage in the firm.

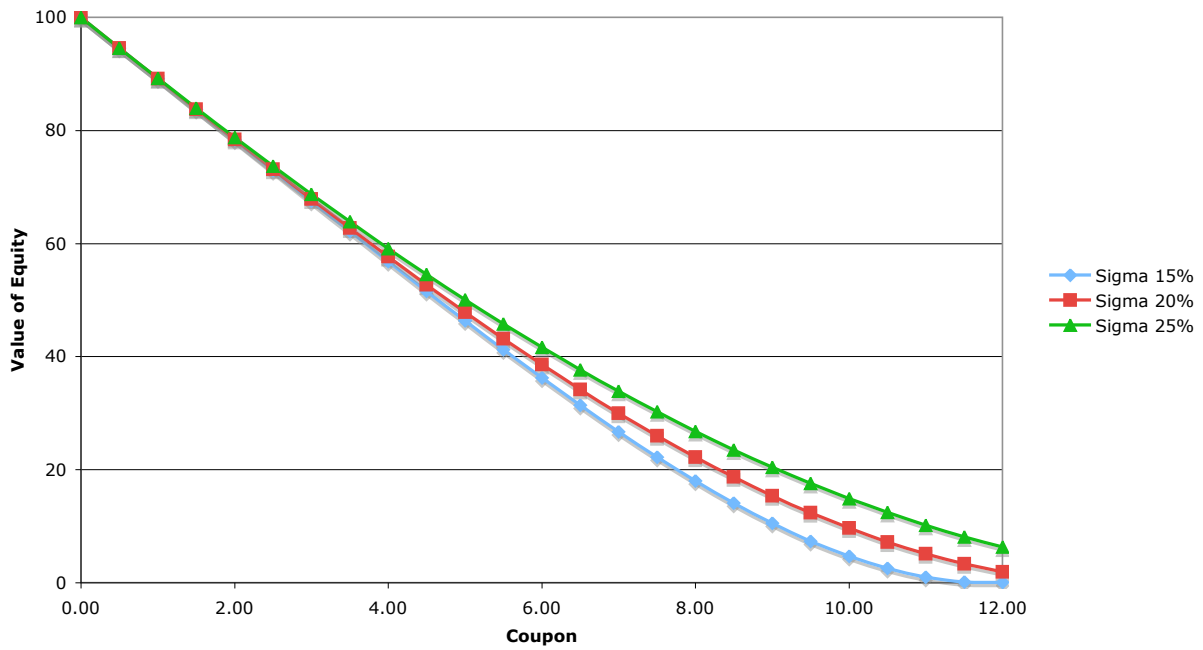


Figure 5: The effect of volatility on the value of equity.

3 Economical Framework

In order to set up a new static capital structure model, the economical framework set up in Broadie et al. (2007) will be used. In addition to the barrier in Leland (1994), a new barrier will now be introduced. The new barrier will be a bankruptcy barrier according to chapter 11. According to the US bankruptcy code, a firm that is not able to pay their debt can file for bankruptcy protection according to chapter 11 in order to escape liquidation. Liquidation is the form of default used in Leland (1994). The bankruptcy barrier will allow for the equity holders to restructure their debt and a grace period where the debt holders are not able to liquidate the firm.

In order to develop the new model in sections 4 and 5, results from Mjøs and Persson (2008) and Mjøs et al. (2008) will be used. These results gives closed form solutions to claims on the firm's cash flow in presence of bankruptcy and liquidation. Also, these expressions uses the same underlying value process as stated in section 2.1, hence these expressions can be implemented without any modifications.

3.1 The Risk of Lost Debt Coupons

In their first article, Mjøs and Persson (2008), the authors provide closed form solutions for claims on the firm's cash flow which are fixed coupon payments. The results are divided into two different sets of solutions. The first set, where a coupon is paid only when the value process, now denoted A , is above the barrier for bankruptcy, denoted A_B . This will be referred to as an above annuity and denoted by subscript A. The second set pays a coupon only when the value process is between the barrier for bankruptcy and the barrier for liquidation, denoted A_L . This claim is referred to as a below annuity and denoted by subscript B. Also, superscript a and b denotes whether the initial value, A_0 , is starting above or below the barrier, respectively. All claims are perpetual claims.

Similar, as in equation (2.16), the price of a claim, denoted U^a , which pays 1 the first time A hits a barrier B from above, is given by

$$U^a = \left[\frac{A}{B} \right]^{-\beta}. \quad (3.1)$$

The price of a claim, denoted U^b , which pays 1 the first time A hit a barrier B from below, is given by

$$U^b = \left[\frac{A}{B} \right]^{\alpha}. \quad (3.2)$$

Going forward, these two expressions will be used for both the price of 1 upon bankruptcy and liquidation. To find the values upon bankruptcy, A_B is substituted in for B . And equivalently, A_L is substituted in for B in order to find the price of 1 upon liquidation.

The value of an above annuity, $V_A^a(A)$, when $A \geq A_B$, is given by

$$V_A^a(A) = \frac{c}{r} \left[1 - \left(\frac{\alpha}{\alpha + \beta} \left(\frac{A_L}{A_B} \right)^{-\beta} + \frac{\beta}{\alpha + \beta} \left(\frac{A_L}{A_B} \right)^\alpha \right) \left(\frac{A}{A_L} \right)^{-\beta} \right]. \quad (3.3)$$

This value is not as intuitive as the annuities in Leland (1994). Here it consist of a risk free perpetual coupon, less a term modeling that the coupon only is paid when $A \geq A_B$ multiplied by the price of 1 upon liquidation.

The value of an above annuity, $V_A^b(A)$, when $A_L \leq A \leq A_B$, is given by

$$V_A^b(A) = \frac{c}{r} \frac{\beta}{\alpha + \beta} \left[\left(\frac{A}{A_B} \right)^\alpha - \left(\frac{A_L}{A_B} \right)^\alpha \left(\frac{A}{A_L} \right)^{-\beta} \right]. \quad (3.4)$$

The interpretation here is similar as for equation (3.3). Perpetual coupon payments if $A \geq A_B$, less the value of losing this possibility if the firm is liquidated.

The value of a below annuity, $V_B^a(A)$, when $A \geq A_B$, is given by

$$V_B^a(A) = \frac{c}{r} \left[\frac{\alpha}{\alpha + \beta} \left(\frac{A_L}{A_B} \right)^{-\beta} + \frac{\beta}{\alpha + \beta} \left(\frac{A_L}{A_B} \right)^\alpha - 1 \right] \left[\frac{A}{A_L} \right]^{-\beta}. \quad (3.5)$$

The value here is gained by coupon payments only when $A_L \leq A \leq A_B$, less the value of losing these coupons if the firm is liquidated.

The value of a below annuity, $V_B^b(A)$, when $A_L \leq A \leq A_B$, is given by

$$V_B^b(A) = \frac{c}{r} \left[1 - \frac{\beta}{\alpha + \beta} \left(\frac{A}{A_B} \right)^\alpha - \left(1 - \frac{\beta}{\alpha + \beta} \left(\frac{A_L}{A_B} \right)^\alpha \right) \left(\frac{A}{A_L} \right)^{-\beta} \right]. \quad (3.6)$$

As for equation (3.5), $V_B^b(A)$ gives the value of coupons paid only when $A_L \leq A \leq A_B$, i.e. when the firm is in bankruptcy.

3.2 Level Dependent Annuities

In their second paper, Mjøs et al. (2008), the authors consider claims with payout depending on the value of the underlying value process. This article is using the same setup as the previous, i.e. the underlying value process is the same as derived in section 2.1.

Again, the infinite annuities in the case with bankruptcy risk will be set up. First, the expressions for claims on a solvent firm will be shown. The value of an above annuity, V_A^a , when $A \geq A_B$, is given by

$$V_A^a = \frac{\lambda A}{r - \mu} - \frac{\lambda A_B}{r - \mu} \left[\frac{\alpha - 1}{\alpha + \beta} \left(\frac{A_L}{A_B} \right)^{-\beta} + \frac{\beta + 1}{\alpha + \beta} \left(\frac{A_L}{A_B} \right)^\alpha \right] \left[\frac{A}{A_L} \right]^{-\beta}. \quad (3.7)$$

Here λ denotes the annuity payment rate. That is, the claim in equation (3.7) pays λA as long as $A \geq A_B$. The value of an above annuity, V_A^b , when $A_L \leq A \leq A_B$, is given by

$$V_A^b = \frac{\lambda A_B}{r - \mu} \frac{\beta + 1}{\alpha + \beta} \left[\frac{A}{A_B} \right]^\alpha - \frac{\lambda A_B}{r - \mu} \frac{\beta + 1}{\alpha + \beta} \left[\frac{A_L}{A_B} \right]^\alpha \left[\frac{A}{A_L} \right]^{-\beta}. \quad (3.8)$$

Similarly, the expressions for claims on a firm in bankruptcy can be set up as follows. The value of a below annuity, V_B^a , when $A \geq A_B$, is given by

$$V_B^a = \frac{\lambda A_B}{r - \mu} \left[\frac{\alpha - 1}{\alpha + \beta} \left(\frac{A_L}{A_B} \right)^{-\beta} + \frac{\beta + 1}{\alpha + \beta} \left(\frac{A_L}{A_B} \right)^\alpha - \frac{A_L}{A_B} \right] \left[\frac{A}{A_L} \right]^{-\beta}. \quad (3.9)$$

Here the value is gained from receiving λA as long as $A_L \leq A \leq A_B$. The value of a below annuity, V_B^b , when $A_L \leq A \leq A_B$, is given by

$$V_B^b = \frac{\lambda A}{r - \mu} - \frac{\lambda A_B}{r - \mu} \frac{\beta + 1}{\alpha + \beta} \left[\frac{A}{A_B} \right]^\alpha - \frac{\lambda A_B}{r - \mu} \left[\frac{A_L}{A_B} - \frac{\beta + 1}{\alpha + \beta} \left(\frac{A_L}{A_B} \right)^\alpha \right] \left[\frac{A}{A_L} \right]^{-\beta}. \quad (3.10)$$

These expressions will be used for setting up the annuities in the new static capital structure.

4 Simple Approach to the New Model

This section will be used to build up a simple approach for the new model. The main purpose of this section will be to show that, when allowing for the barrier for bankruptcy to be set above the barrier for liquidation, value of equity is increased. This is shown by checking the partial derivatives of equity with respect to the barrier for bankruptcy, A_B , and the barrier for liquidation, A_L .

The framework of the new model, will be based on the same claims on the firm's EBIT stream as in Leland (1994). Therefore value of debt, bankruptcy costs, tax benefits, firm, and equity needs to be re-derived using the results from Mjøs and Persson (2008). The underlying value process is still the same as derived in section 2.1. This value process will be denoted A in order to distinguish it from the V in Leland (1994).

4.1 Value of Debt

Let $D(A)$ denote the value of debt in place. The debt promises a perpetual coupon payment, c , as long as the firm is solvent, i.e. when $A \geq A_B$. Here the above annuity from equations (3.3) and (3.4) can be used to model the coupon payments. Also, the terminal value if the firm is liquidated, needs to be added. The terminal value must be multiplied with the probability for liquidation, as in equation (3.1).

The value of debt, when $A \geq A_B$, is therefore given by

$$D^a(A) = \frac{c}{r} \left[1 - \left(\frac{\alpha}{\alpha + \beta} \left(\frac{A_L}{A_B} \right)^{-\beta} + \frac{\beta}{\alpha + \beta} \left(\frac{A_L}{A_B} \right)^{\alpha} \right) \left(\frac{A}{A_L} \right)^{-\beta} \right] + (1 - \lambda_L) A_L \left[\frac{A}{A_L} \right]^{-\beta}. \quad (4.1)$$

The value of debt, when $A_L \leq A \leq A_B$, is given by

$$D^b(A) = \frac{c}{r} \frac{\beta}{\alpha + \beta} \left[\left(\frac{A}{A_B} \right)^{\alpha} - \left(\frac{A_L}{A_B} \right)^{\alpha} \left(\frac{A}{A_L} \right)^{-\beta} \right] + (1 - \lambda_L) A_L \left[\frac{A}{A_L} \right]^{-\beta}, \quad (4.2)$$

where $0 \leq \lambda_L \leq 1$ is the fractional loss in case of liquidation. $D(A)$ is increasing in the parameters c and A , and decreasing in the parameters r and λ_L , for fixed barriers A_B and A_L . Here, the first term gives the value of receiving coupons as long as the firm is solvent. The second term is the terminal value that the debt holders get if the firm is liquidated.

4.2 Value of Liquidation Costs

If the value of the firm reaches the absorbing barrier for liquidation, A_L , some fraction $0 \leq \delta_L \leq 1$ of the firm value is lost in liquidation costs. This loss can be priced by using equation (3.1). Hence, the value of liquidation costs is given by

$$BC_L(A) = \lambda_L A_L \left[\frac{A}{A_L} \right]^{-\beta}. \quad (4.3)$$

$BC_L(A)$ is increasing in the parameter λ_L , and decreasing in the parameter A , for fixed barrier A_L .

4.3 Value of Tax Benefits

The value of the tax benefits is a fraction, $0 \leq \tau \leq 1$, of the coupons paid by the firm. Since the firm only pays coupons when it is solvent, this value is found by using the above annuity from equations (3.3) and (3.4). The value of tax benefits, when $A \geq A_B$, is given by

$$TB^a(A) = \frac{\tau c}{r} \left[1 - \left(\frac{\alpha}{\alpha + \beta} \left(\frac{A_L}{A_B} \right)^{-\beta} + \frac{\beta}{\alpha + \beta} \left(\frac{A_L}{A_B} \right)^{\alpha} \right) \left(\frac{A}{A_L} \right)^{-\beta} \right]. \quad (4.4)$$

Similar, the value of tax benefits, when $A_L \leq A \leq A_B$, is given by

$$TB^b(A) = \frac{\tau c}{r} \frac{\beta}{\alpha + \beta} \left[\left(\frac{A}{A_B} \right)^{\alpha} - \left(\frac{A_L}{A_B} \right)^{\alpha} \left(\frac{A}{A_L} \right)^{-\beta} \right]. \quad (4.5)$$

$TB(A)$ is increasing in the parameters τ , c , and A , and decreasing in the parameter r for fixed barriers A_B and A_L .

4.4 Value of Firm

The total value of the firm, denoted $v(A)$, can be found by adding up the external claims to the firm. For this case, the firm consist of the EBIT stream, the tax benefits, and the liquidation costs. Therefore, the value of the firm, when $A \geq A_B$, is given by

$$\begin{aligned} v^a(A) &= A + TB^a(A) - BC_L(A) \\ &= A + \frac{\tau c}{r} \left[1 - \left(\frac{\alpha}{\alpha + \beta} \left(\frac{A_L}{A_B} \right)^{-\beta} + \frac{\beta}{\alpha + \beta} \left(\frac{A_L}{A_B} \right)^{\alpha} \right) \left(\frac{A}{A_L} \right)^{-\beta} \right] \\ &\quad - \lambda_L A_L \left(\frac{A}{A_L} \right)^{-\beta}. \end{aligned} \quad (4.6)$$

The value of the firm, when $A_L \leq A \leq A_B$, is given by

$$\begin{aligned}
v^b(A) &= A + TB^b(A) - BC_L(A) \\
&= A + \frac{\tau c}{r} \frac{\beta}{\alpha + \beta} \left[\left(\frac{A}{A_B} \right)^\alpha - \left(\frac{A_L}{A_B} \right)^\alpha \left(\frac{A}{A_L} \right)^{-\beta} \right] \\
&\quad - \lambda_L A_L \left(\frac{A}{A_L} \right)^{-\beta}.
\end{aligned} \tag{4.7}$$

$v(A)$ is increasing in the parameters A , τ , and c , and decreasing in the parameters r and λ_L , for fixed barriers A_B and A_L .

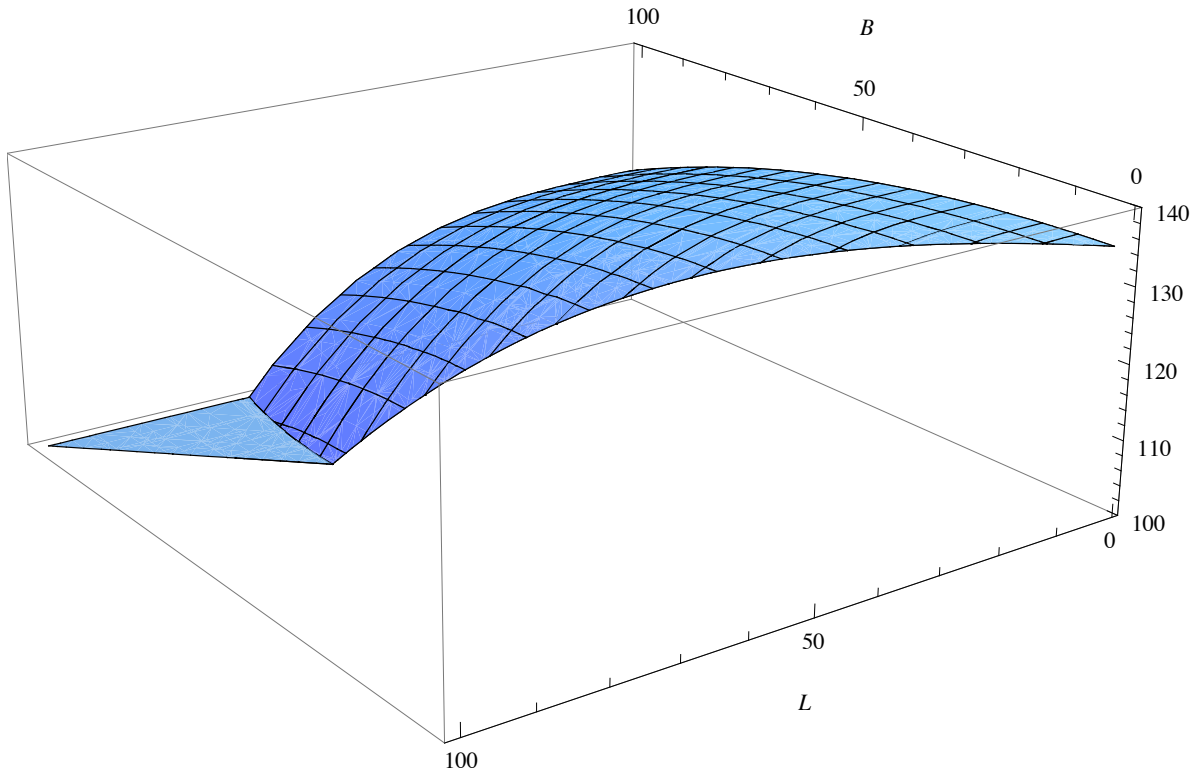


Figure 6: Value of the firm dependent on the barriers for bankruptcy, B , and liquidation, L .

Figure 6 shows how the value of the firm increases as the barriers for bankruptcy and liquidation goes towards zero. Hence, from the total value of the firm's point of view, allowing for early bankruptcy or liquidation is not optimal. The values of the parameters are the same as in section 2.8.

4.5 Value of Equity

According to the absolute priority rule, the equity holders have the residual claim to the firm's EBIT stream. In order to calculate the value of equity, value of debt is subtracted from the value of the firm. Hence, the value of equity, when $A \geq A_B$, is given by

$$\begin{aligned} E^a(A) &= v^a(A) - D^a(A) \\ &= A - \frac{(1-\tau)c}{r} \left[1 - \left(\frac{\alpha}{\alpha+\beta} \left(\frac{A_L}{A_B} \right)^{-\beta} + \frac{\beta}{\alpha+\beta} \left(\frac{A_L}{A_B} \right)^\alpha \right) \left(\frac{A}{A_L} \right)^{-\beta} \right] \\ &\quad - A_L \left(\frac{A}{A_L} \right)^{-\beta}. \end{aligned} \quad (4.8)$$

Similar, the value of equity, when $A_L \leq A \leq A_B$, is given by

$$\begin{aligned} E^b(A) &= v^b(A) - D^b(A) \\ &= A - \frac{(1-\tau)c}{r} \frac{\beta}{\alpha+\beta} \left[\left(\frac{A}{A_B} \right)^\alpha - \left(\frac{A_L}{A_B} \right)^\alpha \left(\frac{A}{A_L} \right)^{-\beta} \right] \\ &\quad - A_L \left(\frac{A}{A_L} \right)^{-\beta}. \end{aligned} \quad (4.9)$$

$E(A)$ is increasing in the parameters A , r , and τ , and decreasing in the parameter c .

From $E(A)$, the cash flows to the equity holders can be identified. The first term is their claim to the firm's EBIT stream, the second term is the after tax deducted coupon payments, and the third term is the loss of the entire firm at liquidation.

4.6 Endogenous Liquidation Barrier

If the equity holders are able to set the barrier for liquidation, they will maximize the value of equity with respect to the barrier, A_L . By using the smooth pasting condition, it is required that the derivative of the process starting above A_B is equal to the derivative of the process starting below A_B . These derivatives are given by

$$\frac{\partial E^a(A)}{\partial A_L} = \frac{(1-\tau)c}{r} \beta A^{-\beta} A_B^{-\alpha} A_L^{\alpha+\beta-1} - (\beta+1) A^{-\beta} A_L^\beta, \quad (4.10)$$

and

$$\frac{\partial E^b(A)}{\partial A_L} = \frac{(1-\tau)c}{r} \beta A^{-\beta} A_B^{-\alpha} A_L^{\alpha+\beta-1} - (\beta+1) A^{-\beta} A_L^\beta. \quad (4.11)$$

From equations (4.10) and (4.11), it is shown that these derivatives will be equal for any value of A_L . Hence, value of equity can be maximized by setting one of these derivatives equal to zero, and solving for A_L .

$$\begin{aligned} \frac{\partial E^a(A)}{\partial A_L} &= 0 \\ \frac{(1-\tau)c}{r} \beta A^{-\beta} A_B^{-\alpha} A_L^{\alpha+\beta-1} - (\beta+1) A^{-\beta} A_L^\beta &= 0. \end{aligned} \quad (4.12)$$

This equation has two possible solutions.

$$\begin{aligned} \text{Either } A_L &= 0, \quad \text{or} \\ A_L &= \left[\frac{(1-\tau)c}{r} \frac{\beta}{\beta+1} \right]^{\frac{1}{1-\alpha}} A_B^{\frac{\alpha}{\alpha-1}}. \end{aligned} \quad (4.13)$$

Here, equity holders might not want to liquidate the firm as long as it has positive value. The reason for this is, that when the firm is in bankruptcy, equity holders pay no coupons. In other word, the only possible cost is the liquidation cost. This is minimized by setting the barrier for liquidation equal to zero.

4.7 Endogenous Bankruptcy Barrier

If the equity holders are able to set the barrier for bankruptcy, they will maximize the value of equity with respect to the barrier, A_B . By using the smooth pasting condition, it is required that the derivative of the process starting above A_B is equal to the derivative of the process starting below A_B . These derivatives are given by

$$\frac{\partial E^a(A)}{\partial A_B} = \frac{(1-\tau)c}{r} \frac{\alpha\beta}{\alpha+\beta} A^{-\beta} A_B^{\beta-1} - \frac{(1-\tau)c}{r} \frac{\alpha\beta}{\alpha+\beta} A^{-\beta} A_L^{\alpha+\beta} A_B^{-\alpha-1}, \quad (4.14)$$

and

$$\frac{\partial E^b(A)}{\partial A_B} = -\frac{(1-\tau)c}{r} \frac{\alpha\beta}{\alpha+\beta} A^{-\beta} A_L^{\alpha+\beta} A_B^{-\alpha-1} + \frac{(1-\tau)c}{r} \frac{\alpha\beta}{\alpha+\beta} A^\alpha A_B^{-\alpha-1}. \quad (4.15)$$

These equations satisfy the smooth pasting condition, only when

$$\begin{aligned} \text{Either } A_B &= A, \quad \text{or} \\ A_B &= 0. \end{aligned} \quad (4.16)$$

Here, equity holders will either go bankrupt straight away, or wait until the value of the EBIT stream is equal to zero.

Figure 7 illustrates how the value of the equity changes with changing barriers for bankruptcy and liquidation. It is evident that equity is maximized when A_B is set as

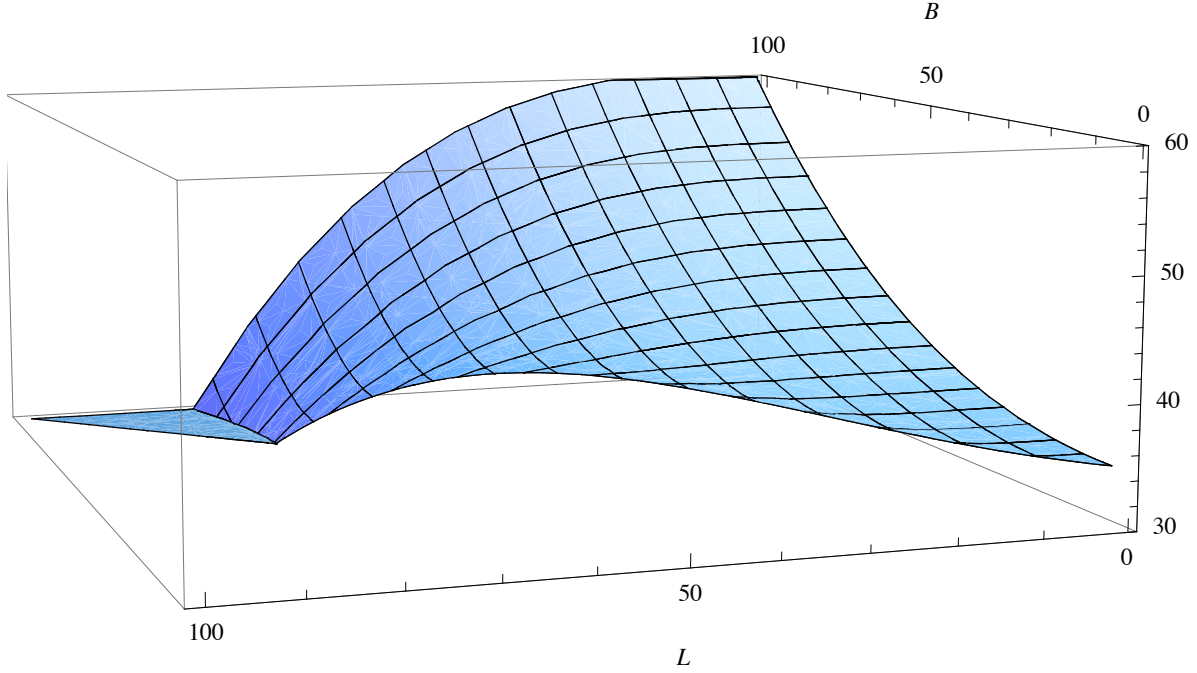


Figure 7: The value of equity dependent on the barriers for bankruptcy, B , and liquidation, L .

high as possible, and A_L is set as low as possible. Hence, from equations (4.13) and (4.16), $A_L = 0$ and $A_B = A$ will be the final solutions to this maximization problem.

The main reason for this extreme solution is that equity holders are offered a free lunch. They can choose to go straight into bankruptcy without taking any costs. In bankruptcy they do not pay any coupons, so as long as there is debt in place, bankruptcy at once will always be most beneficial in this setup.

4.8 Comparative Statics

Here, a new numerical example will be presented, in order to compare the new results from those in section 2.8.

The parameters will again be set to the values; $\mu = 3.5\%$, $\sigma = 20\%$, $r = 6\%$, $\lambda = 50\%$, $\tau = 35\%$, $c = 6$, and $A = 100$.

Now it is necessary to check whether this multi barrier approach can add value to the firm, and/or to the equity in the firm.

Figure 8 is based on non-optimal barriers. The barrier for liquidation is set to the bankruptcy barrier in Leland (1994). The barrier for bankruptcy is set 20% above the barrier for liquidation. The value of the firm remains almost constant as the the model

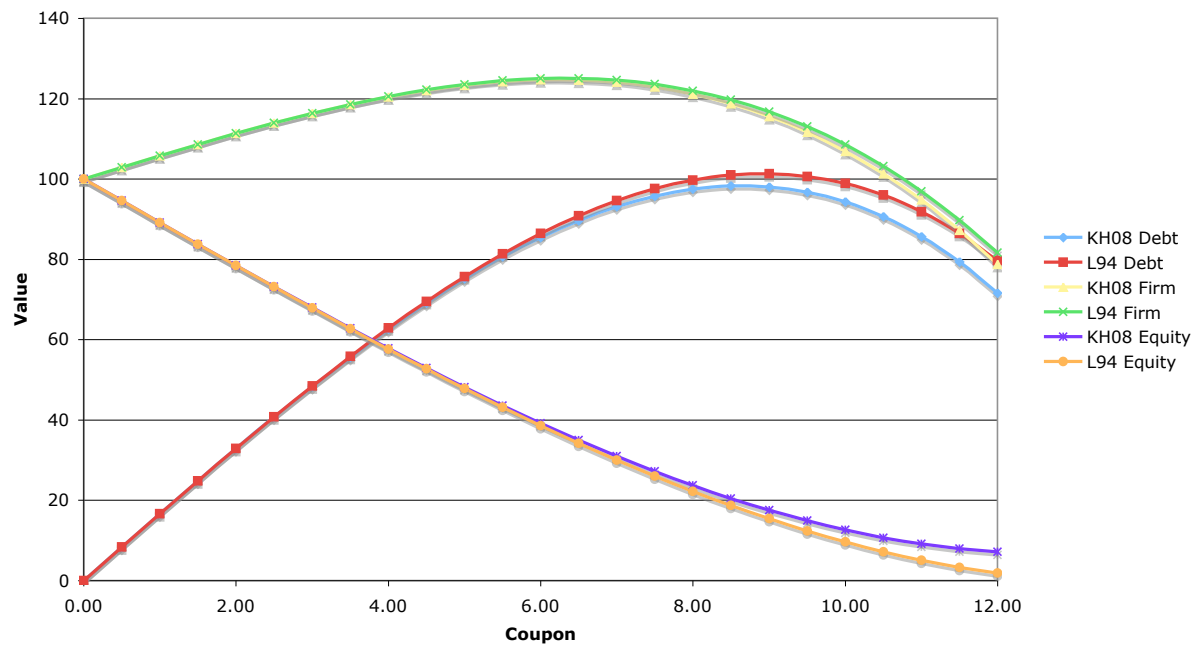


Figure 8: Differences between the two models based on Leland (1994) barrier.

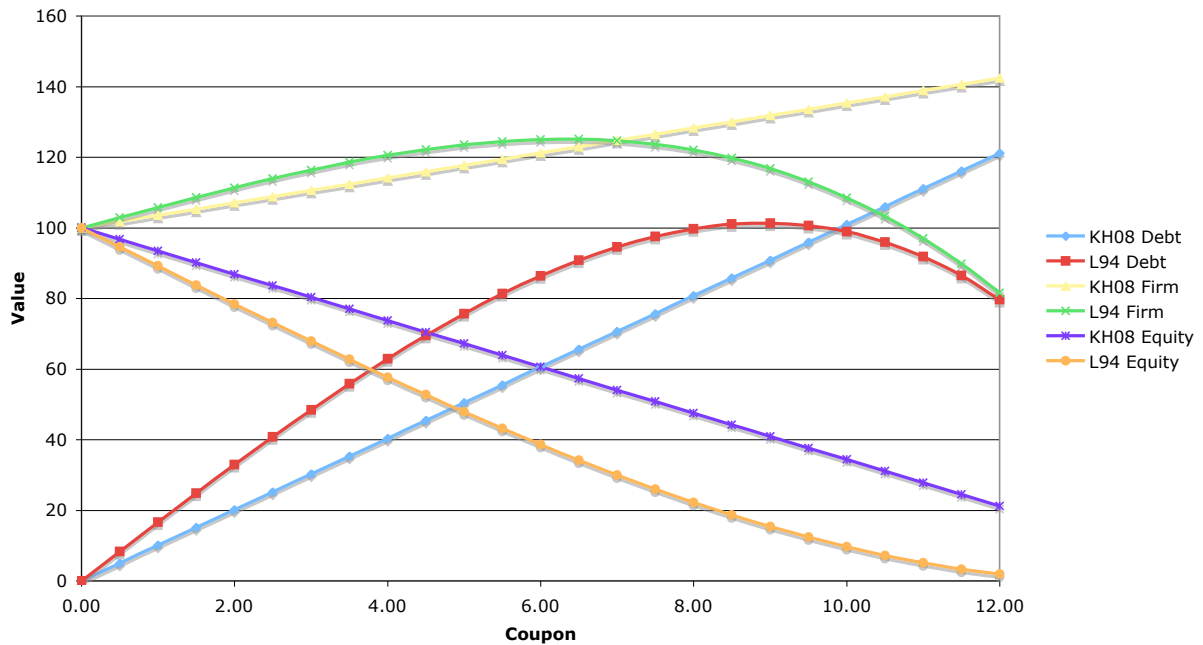


Figure 9: Differences between the two models based on optimal barriers.

change. The slight loss in the new model is due to losing some tax deductions when the firm is in bankruptcy. For the value of debt and for the value of equity the difference is obvious. The main reason for this is that since the coupon is not paid within bankruptcy, this obviously is beneficial for the value of equity. Hence, the value of equity is higher, and the value of debt is lower than in the Leland (1994) model.

Figure 9 is based on the optimal barriers, found from figure 7. The values of debt, firm, and equity, are linear in the coupon. It is obvious that from the equity holders point of view, there is possibly a large gain from using this new multi barrier model. From the debt holders or the total value of the firm's point of view, there is only a gain when the coupon is high, compared to the modified Leland (1994) solution.

Using the economical framework set up in section 3 and 4 illustrates that, when allowing for bankruptcy and liquidation to happen at different levels, equity will have a higher value. Also, for highly levered firms, there might be a higher value of the firm in total. The next section will be used to extend the model presented here. The key issue will be to set the bankruptcy procedure in such a way that the goals of a bankruptcy procedure identified in Hart (2000) are satisfied.

5 New Static Capital Structure Model In Presence of Bankruptcy And Liquidation

In this section, the new static capital structure model will be presented. This model will be based on the model developed in section 4. In order to make the bankruptcy procedure efficient, goals identified by Hart (2000) will be used. First, a good bankruptcy procedure should deliver an ex post efficient outcome. Second, a good bankruptcy procedure should preserve the bonding role of debt by penalizing managers and share holders adequately in bankruptcy states. Third, a good bankruptcy procedure should preserve the absolute priority of claims, except that some portion of value should possibly be reserved for share holders.

The underlying value process presented in section 2.1 will now be slightly modified. Instead of using V , the equation (2.2) will be used to modify all the equations in section 2.1. The annuities provided by Mjøs and Persson (2008) and Mjøs et al. (2008) can be used without any modification, except for a pure notational change from A to δ . Hence, the underlying value will now be the EBIT stream itself, denoted δ . The expressions for α and β will also remain unchanged.

5.1 Value of Debt

Let $D(\delta)$ denote the value of debt in place. The creditors are promised a perpetual coupon payment, c , as long as the firm is solvent. They will receive the firm's EBIT if the firm is in bankruptcy, and the liquidation value if the firm is liquidated. If the firm is able to get out of bankruptcy and regain solvency, the creditors will grant the debtor a cash payment. This cash payment will be referred to as debt forgiveness. The received EBIT's are present due to the second goal identified in Hart (2000). The debt forgiveness will model the third goal. Let δ_B denote the level of the EBIT when the firm declares bankruptcy.

Let δ_L denote the level of the EBIT when the firm is liquidated. In order to set up the expression for the value of debt, we now have four cash flows to identify. First, the coupons, will be an above annuity, as in equations (3.3) and (3.4). These expressions will now be

$$\text{Coupons}^a = \frac{c}{r} \left[1 - \left(\frac{\alpha}{\alpha + \beta} \left(\frac{\delta_L}{\delta_B} \right)^{-\beta} + \frac{\beta}{\alpha + \beta} \left(\frac{\delta_L}{\delta_B} \right)^{\alpha} \right) \left(\frac{\delta}{\delta_L} \right)^{-\beta} \right], \quad (5.1)$$

and

$$\text{Coupons}^b = \frac{c}{r} \frac{\beta}{\alpha + \beta} \left[\left(\frac{\delta}{\delta_B} \right)^\alpha - \left(\frac{\delta_L}{\delta_B} \right)^\alpha \left(\frac{\delta}{\delta_L} \right)^{-\beta} \right]. \quad (5.2)$$

Second, the EBIT's received, when $\delta_L \leq \delta \leq \delta_B$, is a below annuity, as in equations (3.9) and (3.10). Here, the annuity payment rate will be set to 1. These expressions are

$$\text{EBIT}^a = \frac{\delta_B}{r - \mu} \left[\frac{\alpha - 1}{\alpha + \beta} \left(\frac{\delta_L}{\delta_B} \right)^{-\beta} + \frac{\beta + 1}{\alpha + \beta} \left(\frac{\delta_L}{\delta_B} \right)^\alpha - \frac{\delta_L}{\delta_B} \right] \left[\frac{\delta}{\delta_L} \right]^{-\beta}, \quad (5.3)$$

and

$$\text{EBIT}^b = \frac{\delta}{r - \mu} - \frac{\delta_B}{r - \mu} \frac{\beta + 1}{\alpha + \beta} \left[\frac{\delta}{\delta_B} \right]^\alpha - \frac{\delta_B}{r - \mu} \left[\frac{\delta_L}{\delta_B} - \frac{\beta + 1}{\alpha + \beta} \left(\frac{\delta_L}{\delta_B} \right)^\alpha \right] \left[\frac{\delta}{\delta_L} \right]^{-\beta}. \quad (5.4)$$

Third, the liquidation value will be

$$\text{Liq} = (1 - \lambda_L) \frac{\delta_L}{r - \mu} \left[\frac{\delta}{\delta_L} \right]^{-\beta}, \quad (5.5)$$

where $0 \leq \lambda_L \leq 1$ is the fractional liquidation cost.

Finally, fourth, the debt forgiveness, DF, will be modeled as a fraction of the coupons paid to the equity holders, if the firm is in bankruptcy and regains solvency. Hence, equations (3.1) and (3.2) will be used to price this claim. The expressions will be

$$\text{DF}^a = \lambda_B \left[\frac{\delta}{\delta_B} \right]^\alpha \left[\frac{\delta}{\delta_B} \right]^{-\beta} \text{Coupons}^a \quad (5.6)$$

and

$$\text{DF}^b = \lambda_B \left[\frac{\delta}{\delta_B} \right]^\alpha \text{Coupons}^b, \quad (5.7)$$

where $0 \leq \lambda_B \leq 1$ is the fractional forgiveness parameter.

Adding up equations (5.1), (5.3), (5.5), and (5.6), the value of debt, when $\delta \geq \delta_B$, is given by

$$\begin{aligned} D^a(\delta) &= \text{Coupons}^a + \text{EBIT}^a + \text{Liq} - \text{DF}^a \\ &= \left[1 - \lambda_B \left[\frac{\delta}{\delta_B} \right]^\alpha \left[\frac{\delta}{\delta_B} \right]^{-\beta} \right] \frac{c}{r} \left[1 - \left(\frac{\alpha}{\alpha + \beta} \left(\frac{\delta_L}{\delta_B} \right)^{-\beta} + \frac{\beta}{\alpha + \beta} \left(\frac{\delta_L}{\delta_B} \right)^\alpha \right) \left(\frac{\delta}{\delta_L} \right)^{-\beta} \right] \\ &\quad + \frac{\delta_B}{r - \mu} \left[\frac{\alpha - 1}{\alpha + \beta} \left(\frac{\delta_L}{\delta_B} \right)^{-\beta} + \frac{\beta + 1}{\alpha + \beta} \left(\frac{\delta_L}{\delta_B} \right)^\alpha - \frac{\delta_L}{\delta_B} \right] \left[\frac{\delta}{\delta_L} \right]^{-\beta} \\ &\quad + (1 - \lambda_L) \frac{\delta_L}{r - \mu} \left[\frac{\delta}{\delta_L} \right]^{-\beta}. \end{aligned} \quad (5.8)$$

Similarly, equations (5.2), (5.4), (5.5), and (5.7) adds up to the value of debt, when $\delta_L \leq \delta \leq \delta_B$. This is given by

$$\begin{aligned}
D^b(\delta) &= \text{Coupons}^b + \text{EBIT}^b + \text{Liq} - \text{DF}^b \\
&= \left[1 - \lambda_B \left[\frac{\delta}{\delta_B} \right]^\alpha \right] \frac{c}{r} \frac{\beta}{\alpha + \beta} \left[\left(\frac{\delta}{\delta_B} \right)^\alpha - \left(\frac{\delta_L}{\delta_B} \right)^\alpha \left(\frac{\delta}{\delta_L} \right)^{-\beta} \right] \\
&\quad + \frac{\delta}{r - \mu} - \frac{\delta_B}{r - \mu} \frac{\beta + 1}{\alpha + \beta} \left[\frac{\delta}{\delta_B} \right]^\alpha - \frac{\delta_B}{r - \mu} \left[\frac{\delta_L}{\delta_B} - \frac{\beta + 1}{\alpha + \beta} \left(\frac{\delta_L}{\delta_B} \right)^\alpha \right] \left[\frac{\delta}{\delta_L} \right]^{-\beta} \\
&\quad + (1 - \lambda_L) \frac{\delta_L}{r - \mu} \left[\frac{\delta}{\delta_L} \right]^{-\beta}.
\end{aligned} \tag{5.9}$$

5.2 Value of Liquidation Costs

If the value of the EBITs hit the absorbing barrier for liquidation, δ_L , some liquidation costs will occur. The value of these costs can be modeled as the bankruptcy costs in section 2.3. Hence, the value of liquidation costs, BC_L , is given by

$$BC_L(\delta) = \lambda_L \frac{\delta_L}{r - \mu} \left[\frac{\delta}{\delta_L} \right]^{-\beta}. \tag{5.10}$$

$BC_L(\delta)$ is increasing in the parameters λ_L , δ_L , and μ , and decreasing in the parameters r and δ . These costs only occur if the firm is liquidated.

5.3 Value of Tax Benefits

When a firm pays coupons on its debt, it is tax deductible. In other words, the firm will receive an amount equal to the tax rate times the coupon paid. Since the firm only pay coupons when it is solvent, it is again necessary to use the above annuity from equations (3.3) and (3.4) in order to calculate the value of these tax benefits.

The value of tax benefits, when $\delta \geq \delta_B$, is therefore given by

$$TB^a(\delta) = \frac{\tau c}{r} \left[1 - \left(\frac{\alpha}{\alpha + \beta} \left(\frac{\delta_L}{\delta_B} \right)^{-\beta} + \frac{\beta}{\alpha + \beta} \left(\frac{\delta_L}{\delta_B} \right)^\alpha \right) \left(\frac{\delta}{\delta_L} \right)^{-\beta} \right]. \tag{5.11}$$

Similar, when $\delta_L \leq \delta \leq \delta_B$, the value of tax benefits is given by

$$TB^b(\delta) = \frac{\tau c}{r} \frac{\beta}{\alpha + \beta} \left[\left(\frac{\delta}{\delta_B} \right)^\alpha - \left(\frac{\delta_L}{\delta_B} \right)^\alpha \left(\frac{\delta}{\delta_L} \right)^{-\beta} \right]. \tag{5.12}$$

$TB(\delta)$ is increasing in the parameters τ , c , and δ , and decreasing in the parameters r , δ_B , and δ_L . As mentioned in section 2, the firm might not be in a taxable situation. This situation is not taken into consideration for this model.

5.4 Value of Firm

The total value of the firm, denoted $v(\delta)$, is given by three different cash flows in the firm. The EBIT stream, tax benefits, and liquidation costs.

The value of the firm, when $\delta \geq \delta_B$, is therefore found by adding up equations (2.2), (5.11), and (5.10).

$$\begin{aligned}
v^a(\delta) &= \frac{\delta}{r - \mu} + TB^a(\delta) - BC_L(\delta) \\
&= \frac{\delta}{r - \mu} + \frac{\tau c}{r} \left[1 - \left(\frac{\alpha}{\alpha + \beta} \left(\frac{\delta_L}{\delta_B} \right)^{-\beta} + \frac{\beta}{\alpha + \beta} \left(\frac{\delta_L}{\delta_B} \right)^{\alpha} \right) \left(\frac{\delta}{\delta_L} \right)^{-\beta} \right] \\
&\quad - \lambda_L \frac{\delta_L}{r - \mu} \left[\frac{\delta}{\delta_L} \right]^{-\beta}.
\end{aligned} \tag{5.13}$$

Similarly, the value of the firm, when $\delta_L \leq \delta \leq \delta_B$, is found by adding up equations (2.2), (5.12), and (5.10).

$$\begin{aligned}
v^b(\delta) &= \frac{\delta}{r - \mu} + TB^b(\delta) - BC_L(\delta) \\
&= \frac{\delta}{r - \mu} + \frac{\tau c}{r} \frac{\beta}{\alpha + \beta} \left[\left(\frac{\delta}{\delta_B} \right)^{\alpha} - \left(\frac{\delta_L}{\delta_B} \right)^{\alpha} \left(\frac{\delta}{\delta_L} \right)^{-\beta} \right] \\
&\quad - \lambda_L \frac{\delta_L}{r - \mu} \left[\frac{\delta}{\delta_L} \right]^{-\beta}.
\end{aligned} \tag{5.14}$$

$v(\delta)$ is increasing in the parameters δ , μ , τ , and c , and decreasing in the parameters r , λ_L , δ_B , and δ_L .

Figure 10 illustrates the effect the barriers have on the value of the firm. It is obvious that the value of the firm is maximized if the values of these barriers are equal to zero. The reason for this, is that with early bankruptcy, some tax benefits are lost, and with early liquidation, liquidation costs will occur.

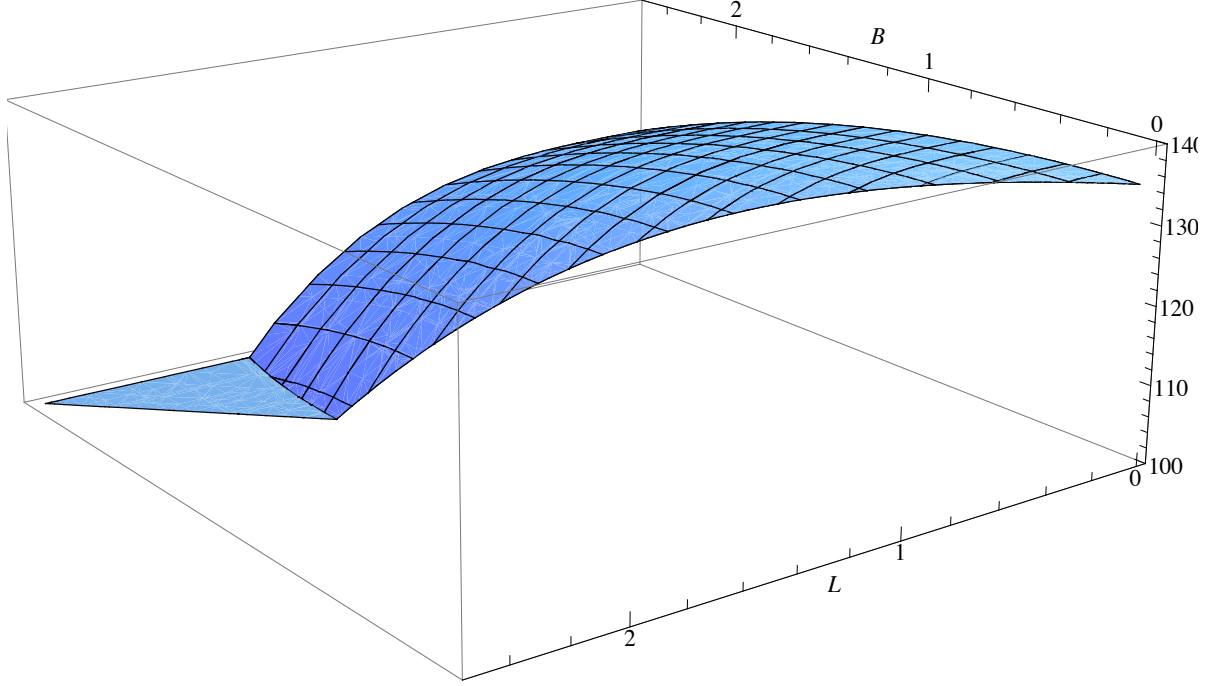


Figure 10: The value of the firm dependent on the barriers for bankruptcy, B , and liquidation, L .

5.5 Value of Equity

The value of equity, denoted $E(\delta)$, is the residual claim on the firm's cash flow. This value can be found by subtracting the value of debt from the value of the firm. Hence, the value of equity, when $\delta \geq \delta_B$, is given by

$$\begin{aligned}
E^a(\delta) &= v^a(\delta) - D^a(\delta) \\
&= \frac{\delta}{r - \mu} + \frac{(1 - \tau)c}{r} \left[\left(\frac{\alpha}{\alpha + \beta} \left(\frac{\delta_L}{\delta_B} \right)^{-\beta} + \frac{\beta}{\alpha + \beta} \left(\frac{\delta_L}{\delta_B} \right)^{\alpha} \right) \left(\frac{\delta}{\delta_L} \right)^{-\beta} - 1 \right] \\
&\quad - \frac{\delta_B}{r - \mu} \left[\frac{\alpha - 1}{\alpha + \beta} \left(\frac{\delta_L}{\delta_B} \right)^{-\beta} + \frac{\beta + 1}{\alpha + \beta} \left(\frac{\delta_L}{\delta_B} \right)^{\alpha} - \frac{\delta_L}{\delta_B} \right] \left[\frac{\delta}{\delta_L} \right]^{-\beta} \\
&\quad + \lambda_B \left[\frac{\delta}{\delta_B} \right]^{\alpha} \left[\frac{\delta}{\delta_B} \right]^{-\beta} \frac{c}{r} \left[1 - \left(\frac{\alpha}{\alpha + \beta} \left(\frac{\delta_L}{\delta_B} \right)^{-\beta} + \frac{\beta}{\alpha + \beta} \left(\frac{\delta_L}{\delta_B} \right)^{\alpha} \right) \left(\frac{\delta}{\delta_L} \right)^{-\beta} \right] \\
&\quad - \frac{\delta_L}{r - \mu} \left[\frac{\delta}{\delta_L} \right]^{-\beta}.
\end{aligned} \tag{5.15}$$

Similar, when $\delta_L \leq \delta \leq \delta_B$, the value of equity is given by

$$\begin{aligned}
E^b(\delta) &= v^b(\delta) - D^b(\delta) \\
&= \frac{\delta}{r - \mu} + \frac{(1 - \tau)c}{r} \frac{\beta}{\alpha + \beta} \left[\left(\frac{\delta_L}{\delta_B} \right)^\alpha \left(\frac{\delta}{\delta_L} \right)^{-\beta} - \left(\frac{\delta}{\delta_B} \right)^\alpha \right] \\
&\quad - \frac{\delta}{r - \mu} + \frac{\delta_B}{r - \mu} \frac{\beta + 1}{\alpha + \beta} \left[\frac{\delta}{\delta_B} \right]^\alpha + \frac{\delta_B}{r - \mu} \left[\frac{\delta_L}{\delta_B} - \frac{\beta + 1}{\alpha + \beta} \left(\frac{\delta_L}{\delta_B} \right)^\alpha \right] \left[\frac{\delta}{\delta_L} \right]^{-\beta} \quad (5.16) \\
&\quad + \lambda_B \left[\frac{\delta}{\delta_B} \right]^\alpha \frac{c}{r} \frac{\beta}{\alpha + \beta} \left[\left(\frac{\delta}{\delta_B} \right)^\alpha - \left(\frac{\delta_L}{\delta_B} \right)^\alpha \left(\frac{\delta}{\delta_L} \right)^{-\beta} \right] - \frac{\delta_L}{r - \mu} \left[\frac{\delta}{\delta_L} \right]^{-\beta}.
\end{aligned}$$

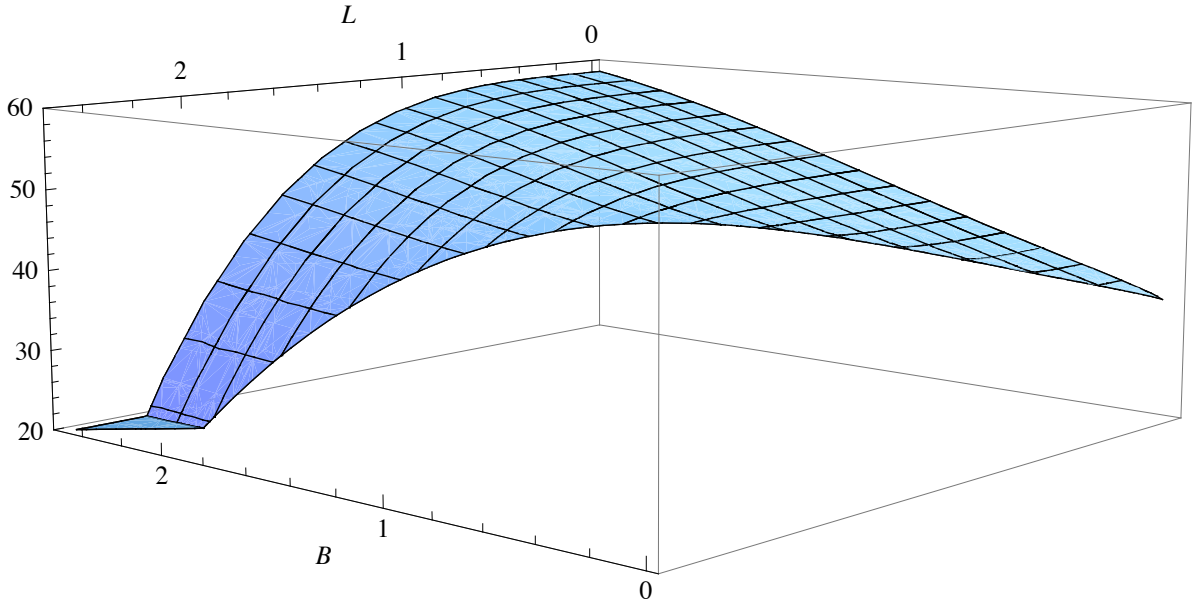


Figure 11: The value of the equity dependent on the barriers for bankruptcy, B , and liquidation, L .

Figure 11 illustrate how the value of equity changes with the barriers for bankruptcy and liquidation. The equity holders in the firm have two options. Either to pay the coupon, or not to pay the coupon. The moment they choose not to pay the coupon triggers bankruptcy. In other words, equity holders are not able to set the liquidation barrier. From the figure we see that equity is maximized when liquidation barrier is equal to zero and the firm goes straight into bankruptcy. However, this is not a plausible outcome. The reason for that is that the creditors have the right to liquidate a firm in chapter 11 after a grace period. The conditions for this grace period can be a given time period that the firm is given to solve their credit issues. Since these thesis is based on perpetual claims, the grace period will be set such that the liquidation barrier is 75% of

the barrier for bankruptcy. This will give the firm some time to resolve their credit issue, and be able to regain solvency.

5.6 Endogenous Bankruptcy Barrier

The equity holders are able to set the bankruptcy barrier by choosing when to stop paying coupons. To find the optimal level for this barrier, we can maximize the value of equity with respect to the barrier, δ_B and setting the derivative equal to zero. This gives

$$\begin{aligned} \frac{\partial E^a(\delta)}{\partial \delta_B} = 0 \\ \frac{(1-\tau)c}{r} \frac{\alpha\beta}{\alpha+\beta} \delta^{-\beta} \left[\delta_B^{\beta-1} - \delta_L^{\alpha+\beta} \delta_B^{-\alpha-1} \right] \\ + \frac{1}{r-\mu} \frac{(\alpha-1)(\beta+1)}{\alpha+\beta} \delta^{-\beta} \left[\delta_L^{\alpha+\beta} \delta_B^{-\alpha} - \delta_B^\beta \right] \\ + \frac{\lambda_{BC}}{r} (\beta-\alpha) \delta^{\alpha-\beta} \delta_B^{\beta-\alpha-1} - \frac{\lambda_{BC}}{r} \frac{\alpha(2\beta-\alpha)}{\alpha+\beta} \delta^{\alpha-2\beta} \delta_B^{2\beta-\alpha-1} \\ - \frac{\lambda_{BC}}{r} \frac{\beta(\beta-2\alpha)}{\alpha+\beta} \delta^{\alpha-2\beta} \delta_L^{\alpha+\beta} \delta_B^{\beta-2\alpha-1} = 0. \end{aligned} \quad (5.17)$$

This expression can be solved numerically for given liquidation barriers. The liquidation barrier is set by the regulations within chapter 11. For the purpose of this model, the barrier for liquidation will be contracted between the creditor and debtor to be 75% of the barrier for bankruptcy. This will allow for the equity to renegotiate their debt, illustrated by the debt forgiveness parameter, and regain solvency.

5.7 Comparative Statics

Using a numerical example of this model, it can be compared to the results from the modified Leland (1994) in section 2.8. The parameters will again be set to the values; $\mu = 3.5\%$, $\sigma = 20\%$, $r = 6\%$, $\lambda = 50\%$, $\tau = 35\%$, $c = 6$, and $\delta = 2.5$ which is equivalent to $V = 100$. Also, as previously mentioned, the $\delta_L = 0.75\delta_B$.

Figure 12 shows how the values of debt, liquidation costs, tax benefits, firm, and equity changes when the coupon changes. The values of debt, tax benefits, and firm are concave in the coupon, while the values of liquidation costs and equity are convex in the coupon. In order to understand these values better, some of the values will be studied more thoroughly, starting off with the value of the firm.

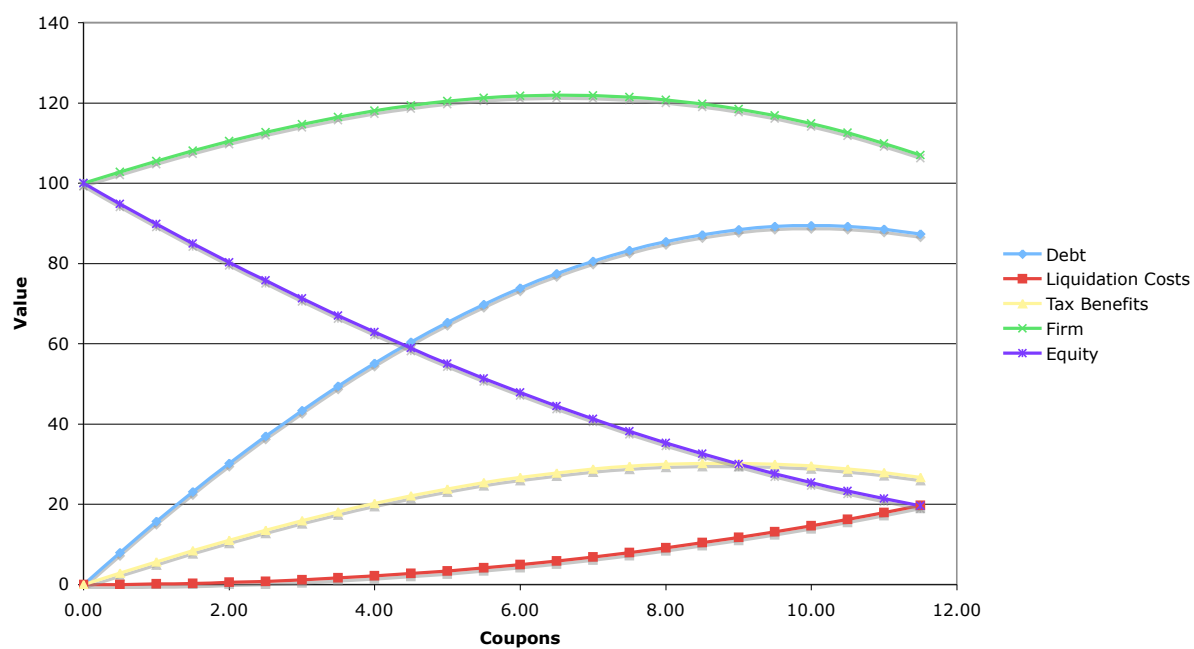


Figure 12: The effect of coupons on the various claims.

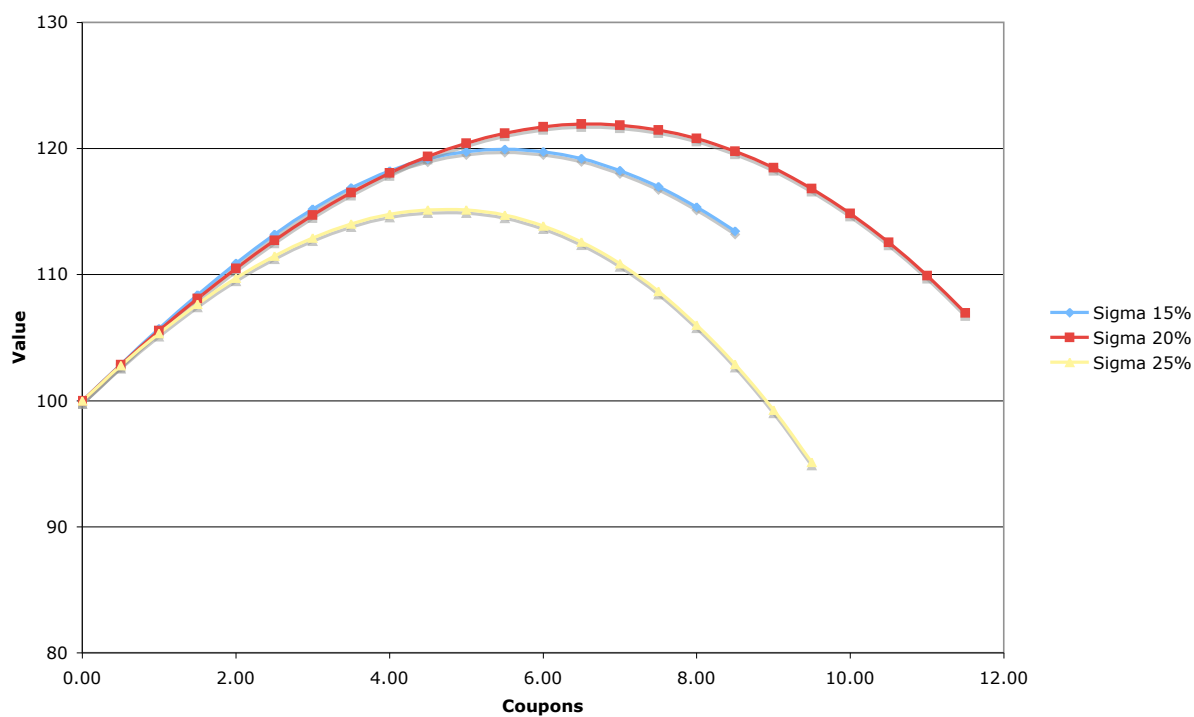


Figure 13: The effect of volatility on the value of the firm.

5.7.1 Value of the Firm

Figure 13 shows how the effect of volatility on the value of the firm. Contrary to Leland (1994), where a low volatility gave the highest value of the firm, the medium volatility gives the highest value. It is clear that the volatility parameter effects the curvature of the plots in both directions. First the curvature flattens, going from 15% to 20% volatility, then it sharpens, going from 20% to 25% volatility. A highly volatile firm optimally puts on coupons of 5, a mid volatile firm optimally puts on coupons of 6.5 and a lowly volatile firm optimally puts on coupons of 5.5. Leland (1994) finds optimal coupons in the range from 6.5 to 7 for all volatilities. From this one can conclude that a firm in the new model is likely to put on less coupons when leveraging the firm, compared to the solutions provided by Leland (1994).

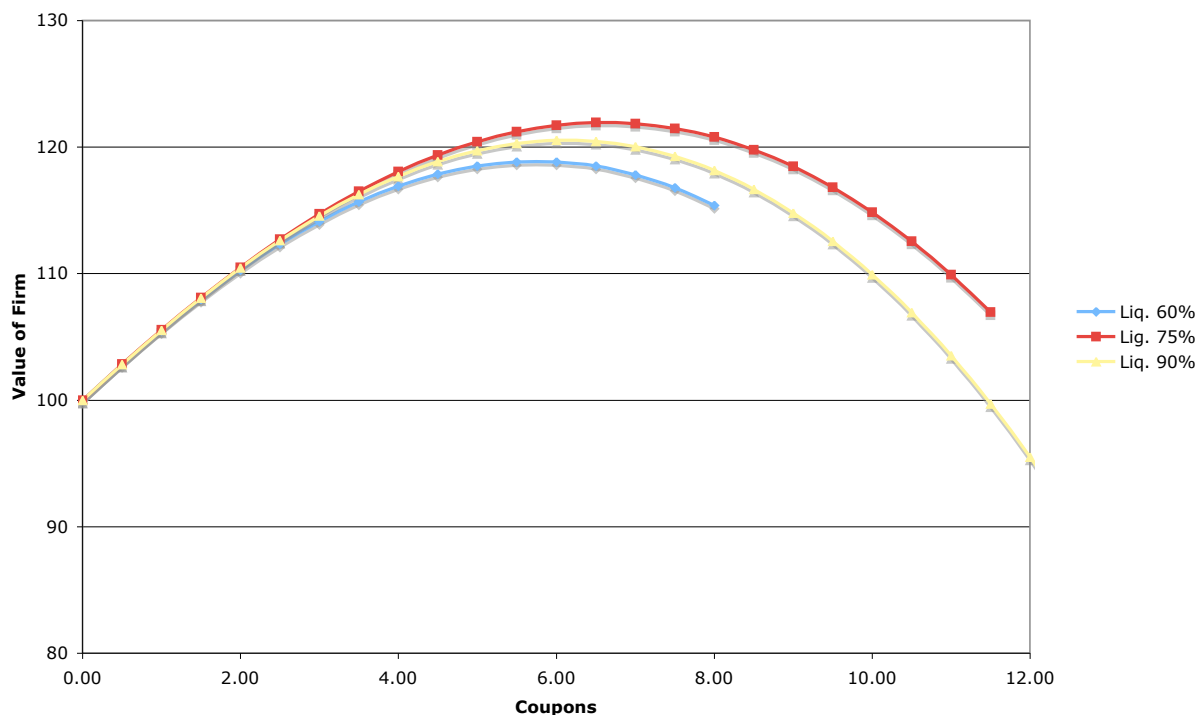


Figure 14: The effect of the liquidation barrier on the value of the firm.

Figure 14 illustrates the effect on the value of the firm allowing for the liquidation barrier to be set at different fractions of the bankruptcy barrier. The mid fraction for liquidation gives the highest value of the firm. Intuitively, it does not make sense that the firm in total will be worth less when the liquidation barrier is set at a lower fraction. However, the reason for this result is that the barriers are set maximizing the value of equity.

Figure 15 illustrates the effect on the value of the firm when the debt forgiveness

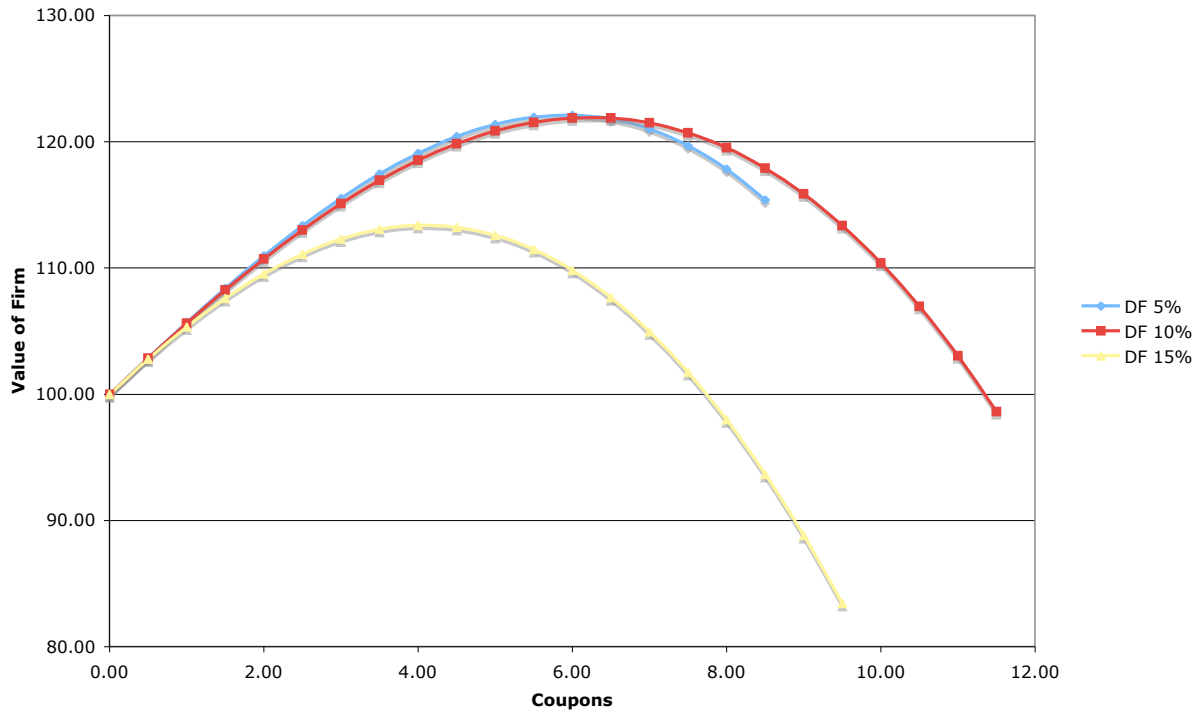


Figure 15: The effect of debt forgiveness on the value of the firm.

factor is changed. Again the results are rather puzzling. Increasing the debt forgiveness parameter from 10% to 15% reduces the value of the firm significantly. The main reason for this is that the equity holders will have good incentives to enter bankruptcy in order to try to regain solvency and get this debt forgiveness. Early bankruptcy results in less tax benefits for the firm. Also worth noticing, is the fact that with high debt forgiveness factor, the firm optimally puts on coupons of only 4.

Figure 16 shows how the value of the firm changes as the leverage changes. Leverage is again calculated by dividing the value of debt by the value of the firm. A high volatile firm is optimally levered at about 45% while the other two illustrated volatilities are optimally levered at about 60-65%. Leland (1994) gives optimal leverage in the range of 70-80% for these volatilities. Hence, for this model, the firm is optimally levered at a lower level.

5.7.2 Value of Equity

Figure 17 shows how the value of equity changes as the coupon changes. Value of equity is decreasing in the coupons, but at a slower pace than in Leland (1994). As equity, in the presence of debt, is an option like instrument, it should be obvious that the value of equity increases with the volatility of the firm's EBIT stream. This is specially shown

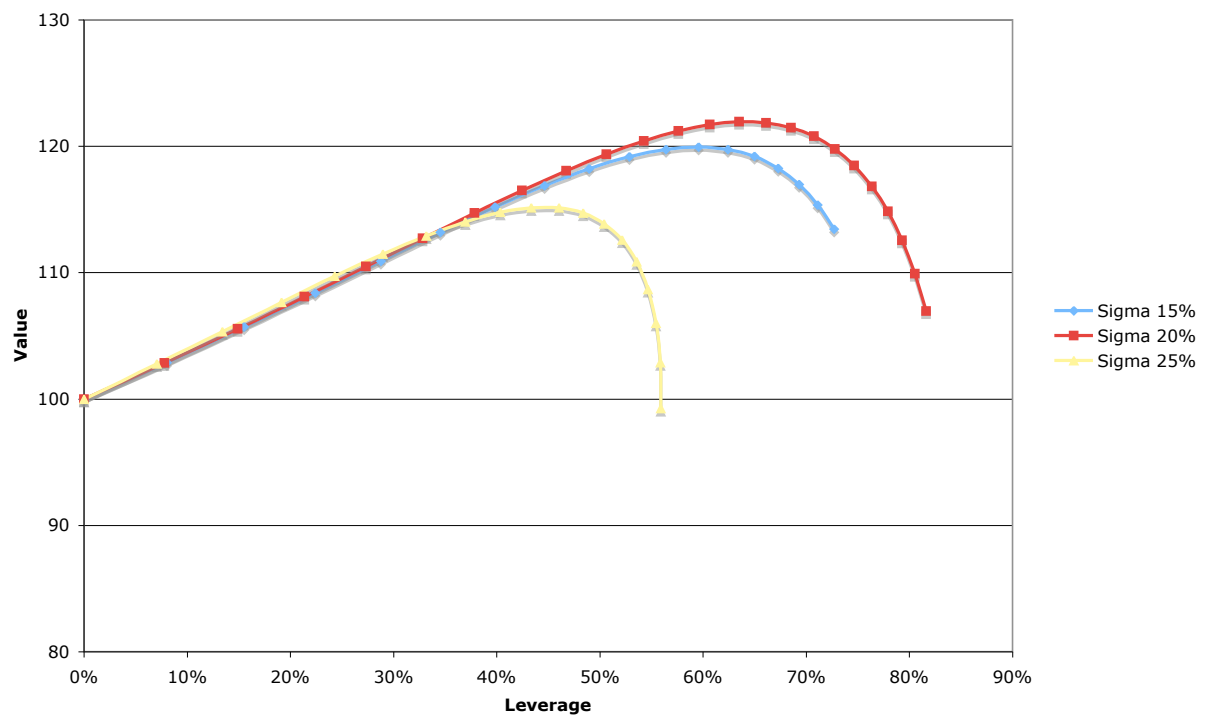


Figure 16: The effect of volatility on the leverage in the firm.

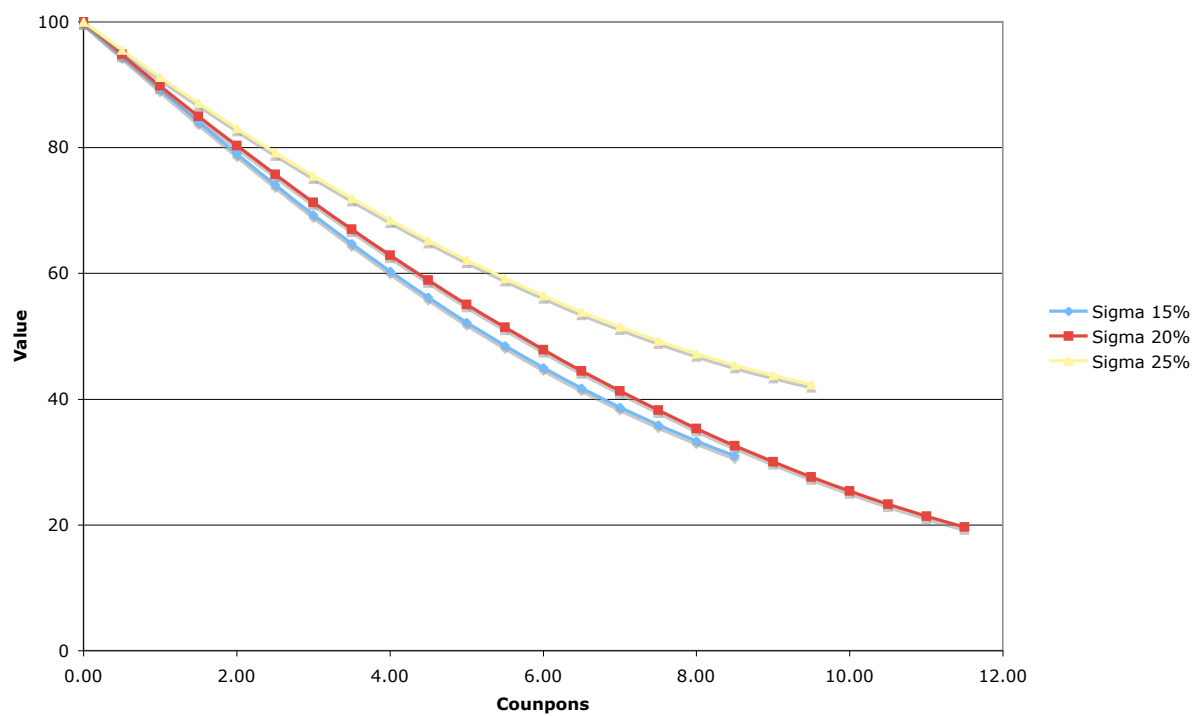


Figure 17: The effect of volatility on the value of equity.

when going to the high volatility, where this curve is much flatter than the other two.

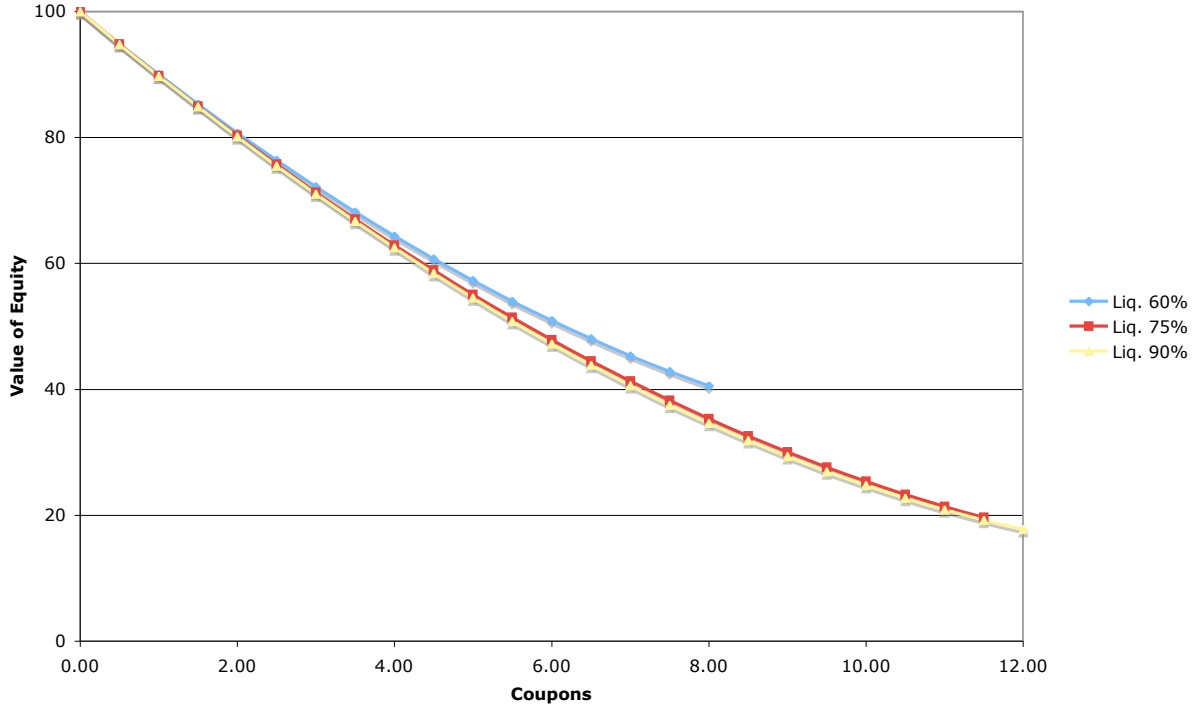


Figure 18: The effect of the liquidation barrier on the value of equity.

Figure 18 shows the effect of changing the gap between the liquidation barrier and the bankruptcy barrier. Going from the base example, i.e. 75%, to 90% does not make much difference for the value of equity. Contrary extending the gap, where liquidation happens at 60% of the lever for bankruptcy, value of equity is increasing significantly.

Figure 19 illustrates the effect of the debt forgiveness parameter. The debt forgiveness is a direct transfer between debt and equity. The observed effect is as expected. Higher λ_B increases the value of equity, and lower λ_B decreases the value of equity.

5.7.3 Barrier for Bankruptcy

Figure 20 shows how the effect of volatility on the bankruptcy barrier. Lower volatility results in earlier bankruptcy than for the base example. For the higher volatility the effect is mixed. Here we get later bankruptcy at low coupons and earlier bankruptcy at high coupons. The main reason for this is probably that the debt forgiveness gets more lucrative and the firm is willing to go into bankruptcy just to bounce out again due to the high volatility. Comparing to the bankruptcy/liquidation barrier in Leland (1994), bankruptcy happens at a earlier stage.

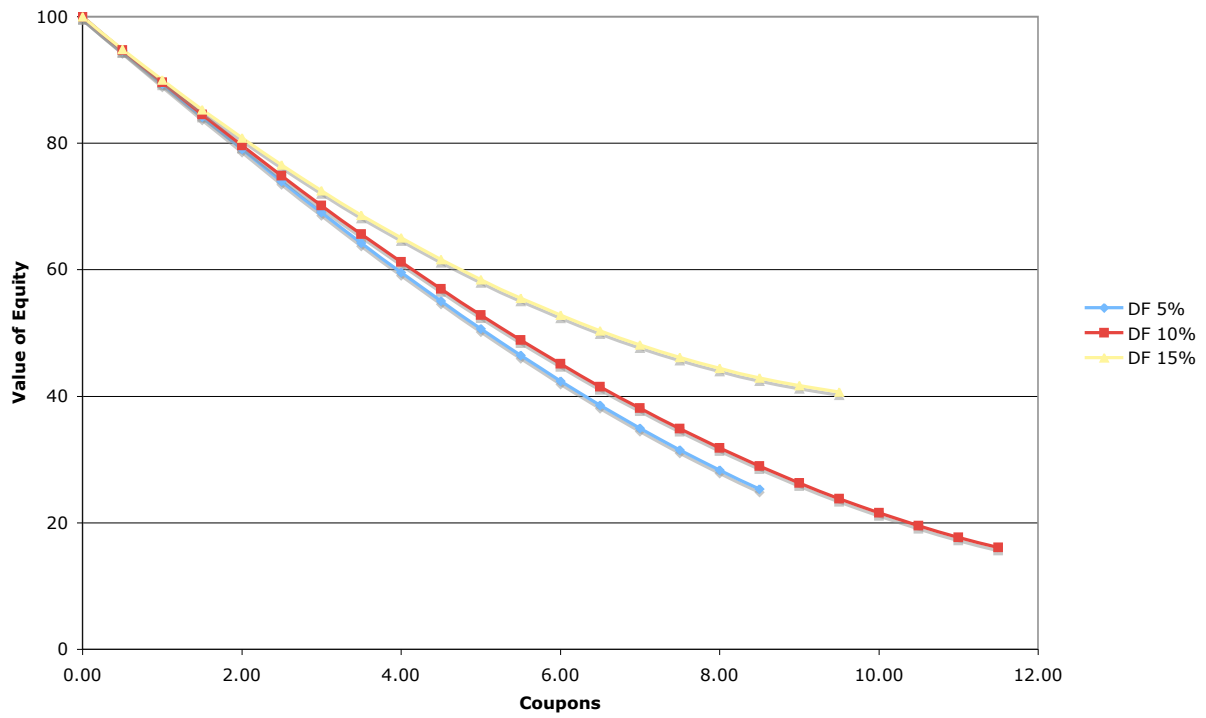


Figure 19: The effect of debt forgiveness on the value of equity.

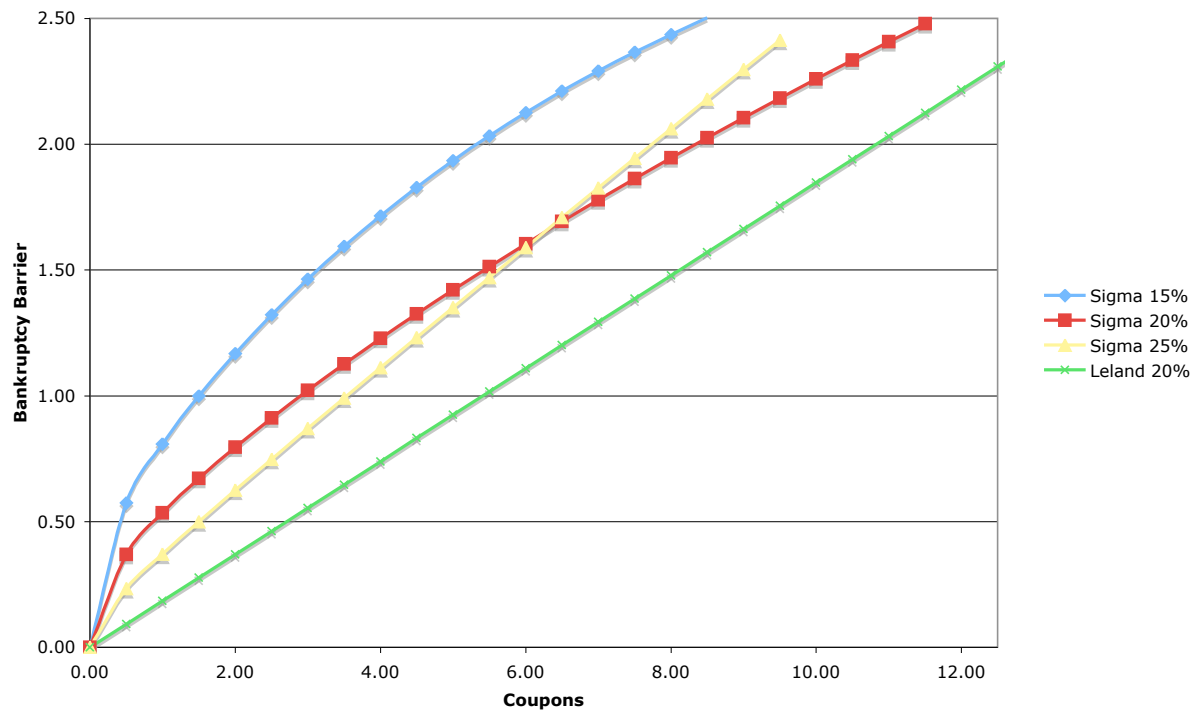


Figure 20: The effect of volatility on the bankruptcy barrier.

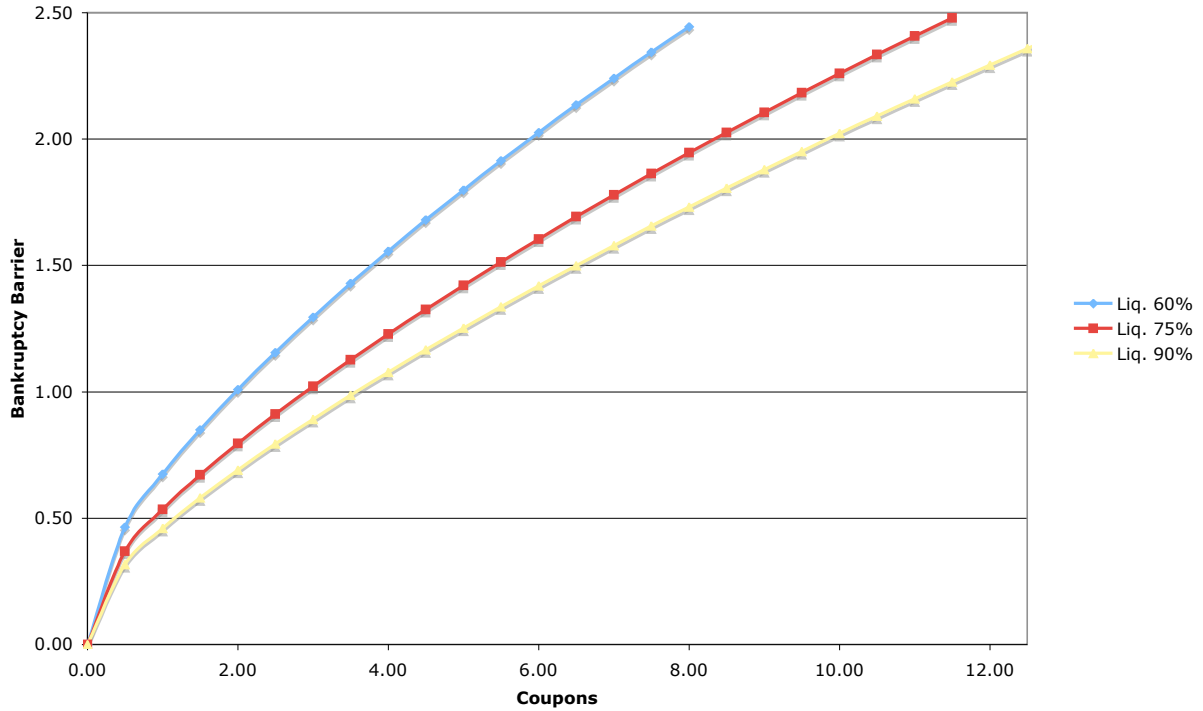


Figure 21: The effect of the liquidation barrier on the bankruptcy barrier.

Figure 21 illustrates the effect of changing the gap between the two barriers. Extending the gap results in earlier bankruptcy, and shortening the gap results in later bankruptcy. An intuitive explanation of this result is that a small gap increases the probability of liquidation. Hence, the equity holders are not willing to try to get the debt forgiveness by deliberately going into bankruptcy just to get out again as soon as possible.

Figure 22 illustrates the effect of the debt forgiveness factor on the bankruptcy barrier. Making the debt forgiveness larger, i.e. increasing λ_B , makes again the bankruptcy state more attractive. The reason that there is almost no effect of going from 5% to 10% is that in the bankruptcy state, the equity holders lose both their EBIT stream and the tax benefits. Therefore the debt forgiveness must be larger than the losses in order to attract equity holders to go for the debt forgiveness.

5.7.4 Leland (1994)

Concluding this section, some comparison to the Leland (1994) should be done. Figures 23 and 24 illustrate the differences between the new model and Leland (1994) for mid and high volatility. The effect of high versus mid or low volatility on the firm's capital structure is completely opposite for the two models. Going from mid to high volatility

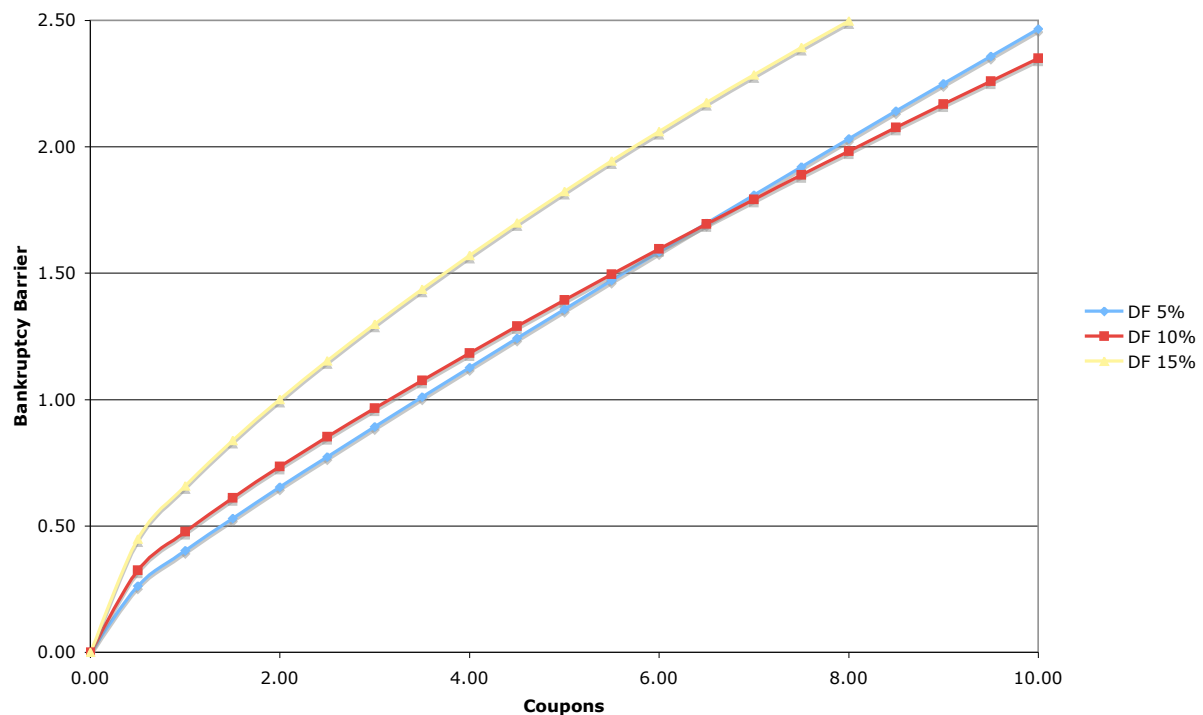


Figure 22: The effect of debt forgiveness on the bankruptcy barrier.

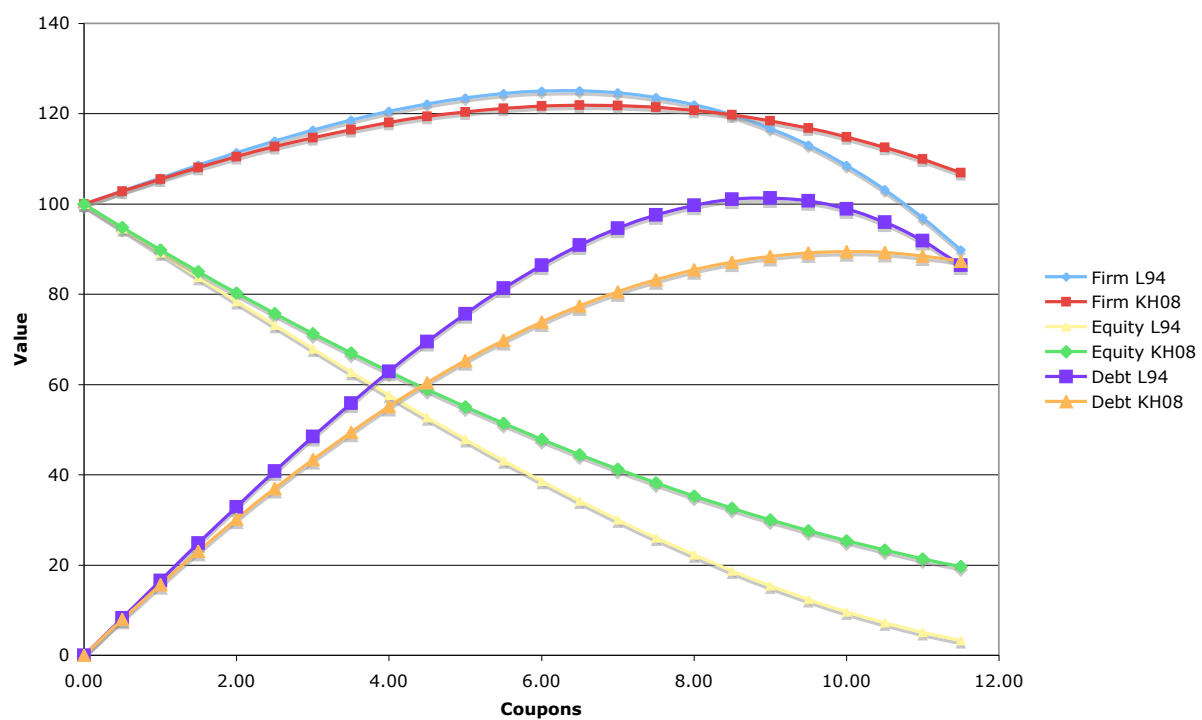


Figure 23: Differences between Leland (1994) and the new model. Volatility at 20%

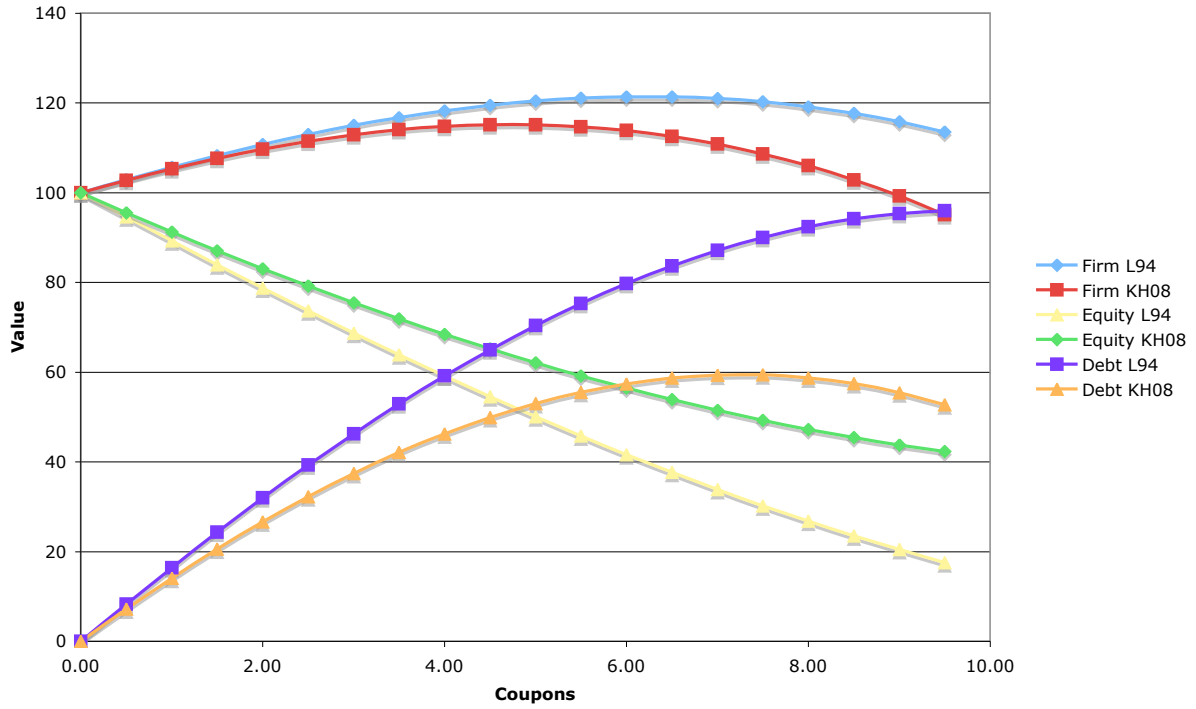


Figure 24: Differences between Leland (1994) and the new model. Volatility at 25%.

increases the amount of debt in the Leland (1994) model. On the contrary, in the new model this reduces the amount of debt added to the firm's capital structure. The exact same effect is observed for the value of debt. For the value of equity, the effect of going from mid to high volatility is the same for both models. But the value of equity is a bit higher in the new model compared to Leland (1994).

6 Conclusions

This thesis has focused on using the framework provided in Broadie et al. (2007) to extend the Leland (1994) model. Doing so, valuations of claims on the firm's cash flow have been set according to annuities set forth in Mjøs and Persson (2008) and Mjøs et al. (2008). The bankruptcy procedure has been set according to the goals identified in Hart (2000).

The new static model for capital structure is based on equity holders choosing when to enter bankruptcy by stopping to pay coupons. The creditors have been locked by a contract, so they can not choose when they want the firm liquidated. This is similar to the grace period used in Broadie et al. (2007). A interesting study would be to check if there exists an equilibrium, where liquidation is below bankruptcy, if the creditors are given the right to liquidate the firm whenever the firm is in bankruptcy.

The results provided by the new model gives useful insights to how different parameters effect the firm's capital structure and the barriers for bankruptcy and liquidation. Compared to the benchmark model, some parameters behave similar, while other gives effects that are very different between the models. Comparing the actual outcomes, the new model transfer value from the creditors to the equity holders without changing the total value of the firm substantially. In other words, optimal level of leverage is reduced towards levels that can be found empirically.

There are some limitations to the model. First of all, it is a static model. That means that the capital structure in the firm is never re-optimized. The firm is levered once, and then leverage moves as the value of the EBIT's move. Second, it does not take into account whether the firm actually is in a taxable position or not. Tax benefits are treated as a payout from the government. Third, the first goal identified by Hart (2000) might not be satisfied. As long as the firm is liquidated when $A \geq 0$, the bankruptcy procedure may not be ex post efficient. However, this can be solved by selling the firm as a going firm and not selling off the individual assets in the firm.

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